DIY: *Exponent Basics*

To review the meaning of exponents and the rules for combining numbers with exponents, watch the following set of YouTube videos. (Note for your security: all hyperlink addresses should begin with https://youtube.com/watch?v=) They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. https://www.youtube.com/watch?v=-zUmvpkhvW8 Introduction to exponents
2. https://www.youtube.com/watch?v=S3IEeCyUWWA Exponents in Algebra
3. https://www.youtube.com/watch?v=g5ZGDNxJwxA Laws of exponents

   Correction: at 2:20, there is a typo in the third line. $x^2 = 1 \cdot x$ should read $x^1 = 1 \cdot x$
4. https://www.youtube.com/watch?v=wsaH5CARIHI Using laws of exponents, part 1
5. https://www.youtube.com/watch?v=X72qoK6i2B8 Using laws of exponents, part 2

(**Note**: for more on this topic, see the DIY: Simplifying Exponential Expressions)

Practice Problems: (assume all variables represent positive numbers)

1. a. The expression $5 \cdot 5 \cdot 5 \cdot 5$ = _____ in exponential form.

   b. 6^3 could be written without an exponent as _____

   c. The expression $(x-5)^2$ = _____ (written without exponents)

   Note: $\boldsymbol{(x-5)^2 \neq x^2 - 5^2}$

   d. $x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z \cdot z =$ _____ in exponential form.

2. a. $3^4 =$ _____ b. $3^0 =$ _____ c. $3^{-4} =$ _____ d. $\left(\frac{2}{3}\right)^{-2} =$ _____

3. a. $(x^6)(x^4) =$ _____ b. $(x^5)(y^5) =$ _____ c. $(b^7)(b^{-3}) =$ _____ d. $(a^3)(b^5) =$ _____

4. a. $\frac{x^{10}}{x^3} =$ _____ b. $\frac{x^5}{y^5} =$ _____ c. $\frac{t^3}{t^{-4}} =$ _____ d. $\frac{a^4}{a^9} =$ _____

5. a. $x^3 + x^3 =$ _____ b. $7b^4 - b^4 =$ _____ c. $3x^2 + 5x^3 + 6x^2 - 9x^3 =$ _____

6. a. $x^3 + y^3 =$ _____ b. $\frac{a^3b^{-2}c^7}{a^4b^{-6}c^2} =$ _____ c. $\left(\frac{t^{-5}z^{14}}{t^{-18}z^{-5}}\right)^0 =$ _____

7. a. $(-3)^4 =$ _____ b. $-3^4 =$ _____ c. $5(xy)^{-3} =$ _____ d. $(5xy)^{-3} =$ _____

8. a. $(3^2)^3 =$ _____ b. $(x^2)^3 =$ _____ c. $\left(\frac{5}{x^4}\right)^{-2} =$ _____ d. $\frac{(6p^5)^3}{(3p^3)^4} =$ _____

Answers

1. a. 5^4 b. $6 \cdot 6 \cdot 6$ c. $(x-5)(x-5) = x^2 - 10x + 25$ d. $x^3y^2z^3$

2. a. 81 b. 1 c. $\frac{1}{81}$ d. $\frac{9}{4}$

3. a. x^{10} b. $(xy)^5$ c. b^4 d. a^3b^5 (cannot be simplified further)

4. a. x^7 b. $\left(\frac{x}{y}\right)^5$ c. t^7 d. $a^{-5} = \frac{1}{a^5}$

5. a. $2x^3$ b. $6b^4$ c. $9x^2 - 4x^3$

6. a. $x^3 + y^3$ (cannot be simplified) b. $\frac{b^4c^5}{a}$ c. 1

7. a. 81 b. -81 c. $\frac{5}{x^3y^3}$ d. $\frac{1}{125x^3y^3}$ 8. a. $3^6 = 729$ b. x^6 c. $\frac{x^8}{25}$ d. $\frac{8p^3}{3}$

Detailed Solutions

1. a. $5 \cdot 5 \cdot 5 \cdot 5 = 5^4$ (1 2 3 4) b. $6^3 = 6 \cdot 6 \cdot 6$ or 216

c. $(x-5)^2 = (x-5)(x-5) = x^2 - 5x - 5x + 25 = x^2 - 10x + 25$
$\neq x^2 - 5^2$

d. $x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z \cdot z = x^3 y^2 z^3$ (1 2 3 1 2 1 2 3)

2. a. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 = \boxed{81}$ b. $3^0 = \boxed{1}$ (any number)$^0 = 1$ except 0^0 is undefined

c. $3^{-4} = \frac{1}{3^4} = \boxed{\frac{1}{81}}$ d. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

or $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{(2^2)^{-1}}{(3^2)^{-1}} = \frac{4^{-1}}{9^{-1}} = \frac{9}{4}$

3. a. $x^6 \cdot x^4 = x^{6+4} = \boxed{x^{10}}$ $x^6 \cdot x^4 = \underbrace{\underbrace{x \cdot x \cdot x \cdot x \cdot x \cdot x}_{6 x's} \cdot \underbrace{x \cdot x \cdot x \cdot x}_{4 x's}}_{10 x's.}$

b. $(x^5)(y^5) = x^5 y^5 = (xy)^5$ The bases are different so the exponents cannot be combined.

c. $b^7 \cdot b^{-3} = b^{7-3} = \boxed{b^4}$ or $b^7 \cdot b^{-3} = \frac{b^7}{b^3} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = b^4$

d. $(a^3)(b^5) = a^3 b^5$ These cannot be combined since the bases are different.

4. a. $\frac{x^{10}}{x^3} = x^{10-3} = \boxed{x^7}$ or $\frac{x \cdot x \cdot x \cdot \overbrace{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}^{7 x's.}}{x \cdot x \cdot x} = x^7$

4.b. $\frac{x^5}{y^5}$ the bases are different so the exponents cannot be combined, but since the exponents are the same, this could be written as $\left(\frac{x}{y}\right)^5$

c. $\frac{t^3}{t^{-4}} = t^{3-(-4)} = t^{3+4} = \boxed{t^7}$ or $\frac{t^3}{t^{-4}} = t^3 \cdot t^4 = t^{3+4} = t^7$

d. $\frac{a^4}{a^9} = a^{4-9} = a^{-5} = \boxed{\frac{1}{a^5}}$ (for simplest form, the answer usually should not contain negative exponents.)

or $\frac{a^4}{a^9} = \frac{a \cdot a \cdot a \cdot a \cdot 1}{a \cdot a \cdot a \cdot a \cdot \underbrace{a \cdot a \cdot a \cdot a \cdot a}_{5\ a's}} = \frac{1}{a^5}$

5. a. $x^3 + x^3 = \boxed{2x^3}$ (Similar to 1 apple + 1 apple = 2 apples).

$= 1x^3 + 1x^3 = (1+1)x^3$

$= 2x^3$

When adding variables with exponents, they are "like terms" if both the bases and the exponents are the same. Then, add the coefficients.

b. $7b^4 - b^4 = (7-1)b^4 = \boxed{6b^4}$

c. $3x^2 + 5x^3 + 6x^2 - 9x^3 = 3x^2 + 6x^2 + 5x^3 - 9x^3$

(like terms; like terms)

$= \boxed{9x^2 - 4x^3}$

6. a. $x^3 + y^3$ cannot be simplified since they are <u>not</u> like terms.

Note: $x^3 + y^3 \neq (x+y)^3$

← this would be $(x+y)(x+y)(x+y)$

6.b. $\dfrac{a^3 b^{-2} c^7}{a^4 b^{-6} c^2} = (a^{3-4})(b^{-2-(-6)})(c^{7-2})$

$= a^{-1} b^{-2+6} c^5 = a^{-1} b^4 c^5 = \boxed{\dfrac{b^4 c^5}{a}}$

c. $\left(\dfrac{t^{-5} z^{14}}{t^{-18} z^{-5}}\right)^0$ this would be time-consuming to simplify the inside, but since (anything)$^0 = 1$, there is no need to simplify. $(\ )^0 = \boxed{1}$

7.a. $(-3)^4 = (-3)(-3)(-3)(-3) = \boxed{81}$ b. $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = \boxed{-81}$

c. $5(xy)^{-3} = \dfrac{5}{(xy)^3} = \boxed{\dfrac{5}{x^3 y^3}}$ d. $(5xy)^{-3} = \dfrac{1}{(5xy)^3} = \dfrac{1}{5^3 x^3 y^3}$

$= \boxed{\dfrac{1}{125 x^3 y^3}}$

8. a. $(3^2)^3 = 9^3 = 9 \cdot 9 \cdot 9 = \boxed{729}$

or $(3^2)^3 = 3^{2 \cdot 3} = 3^6 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 \cdot 9 = 729$

b. $(x^2)^3 = x^{2 \cdot 3} = \boxed{x^6}$

c. $\left(\dfrac{5}{x^4}\right)^{-2} = \dfrac{5^{-2}}{(x^4)^{-2}} = \dfrac{5^{-2}}{x^{-8}} = \dfrac{x^8}{5^2} = \boxed{\dfrac{x^8}{25}}$

or $= \left(\dfrac{x^4}{5}\right)^2 = \dfrac{(x^4)^2}{5^2} = \dfrac{x^{4 \cdot 2}}{25} = \dfrac{x^8}{25}$

d. $\dfrac{(6p^5)^3}{(3p^3)^4}$ because the exponents on numerator and denominator are different, we cannot first simplify the inside expressions

~~$\dfrac{6p^5}{3p^3} = 2p^2$~~

$= \dfrac{6^3 (p^5)^3}{3^4 \cdot (p^3)^4} = \dfrac{(2 \cdot 3)^3 p^{5 \cdot 3}}{3^4 p^{3 \cdot 4}} = \dfrac{2^3 \cdot 3^3 \cdot p^{15}}{3^4 \cdot p^{12}} = 2^3 \cdot 3^{3-4} \cdot p^{15-12} = 2^3 \cdot 3^{-1} \cdot p^3$

$= \boxed{\dfrac{8p^3}{3}}$

Additional Resources

1. https://www.kutasoftware.com/freeipa.html Choose the section entitled "Exponents and Radicals",
    - Multiplication property of exponents
    - Division property of exponents
    - Powers of products and quotient
    - Square roots (optional)

2. https://www.kutasoftware.com/free.html Choose the section entitled "Exponents", subtitle "Properties of exponents".

3. https://www.khanacademy.org Click on "Courses" in the upper left corner. Under the Math column, choose Algebra 1, then Rational exponents and radicals. Choose Exponent properties review.