DIY: *Polynomial Long Division, Synthetic Division*

To review Polynomial Long Division, watch the following set of YouTube videos introducing a review of long division, division by a monomial, long division of a polynomial by another polynomial, and synthetic division. The videos are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. https://www.youtube.com/watch?v=txEyF2QzCqs (review of long division)
2. https://www.youtube.com/watch?v=nv_djIs0v2U (dividing by a monomial)
3. https://www.youtube.com/watch?v=Ih1wb6AxhMI (dividing by a binomial)
4. https://www.youtube.com/watch?v=FTRDPB1wR5Y (dividing polynomials)
   Note that this presenter is locating terms in the quotient slightly differently than the presenter in Video 3. Both methods are correct.
5. https://www.youtube.com/watch?v=lLgRS0mUZLw (synthetic division)

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. a. Simplify: $\frac{3x^5}{2x^3} =$ __________ b. Simplify: $\frac{4x^7}{8x^2} =$ __________

2. Divide: $\frac{6x^4+9x^3-3x^2+4x+1}{3x^2}$

3. Divide: $(x^4 + 3x^3 - 2x^2 - 3x + 1) \div (x + 2)$

4. $\frac{x^3-3x^2-4x+12}{x-3} =$ ____________

5. Divide: $\frac{8x^3+2x^2+1}{2x^2-x}$

6. Use long division: $\frac{x^6-64}{x-2}$

7. Divide: $\frac{2x^4+x^3+x-\frac{3}{4}}{x-\frac{1}{2}}$

8. Divide: $\frac{4x^5+13x^4+108x^2-81}{x^2+9}$

9. Find $(5x^4+5x^2+5) \div (x^2-x+1)$

10. The difference of cubes factoring pattern can be easily forgotten, but can be derived by doing the following long division problem: $\frac{a^3-b^3}{a-b} =$ ________________ . So $a^3-b^3=(a-b)($________________$)$

11. Redo the division in Question #3 using synthetic division.

12. Redo Question #6 using synthetic division.

Answers:

1. a. $\frac{3}{2}x^2$ b. $\frac{x^5}{2}$

2. $2x^2+3x-1+\frac{4x+1}{3x^2}$

3. $x^3+x^2-4x+5-\frac{9}{x+2}$

4. x^2-4 (remainder = 0)

Note: Since $\frac{x^3-3x^2-4x+12}{x-3} = x^2-4$, then $x^3-3x^2-4x+12=(x-3)(x^2-4)$.

$(x-3)$ is a **factor** of $x^3-3x^2-4x+12$.

5. $4x+3+\frac{3x+1}{2x^2-x}$

6. $x^5+2x^4+4x^3+8x^2+16x+32$

7. $2x^3+2x^2+x+\frac{3}{2}$

8. $4x^3+13x^2-36x+9-\frac{324x}{x^2+9}$

9. $5x^2+5x+5$

10. (a^2+ab+b^2)

11. (same as Q. 3)

12. (same as Q. 6)

Detailed Solutions

1.a $\frac{3x^5}{2x^3} = \frac{3\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}{2\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = \boxed{\frac{3}{2}x^2}$

b. $\frac{4x^7}{8x^2} = \frac{\overset{1}{\cancel{4}}\cdot\overset{x^5}{\cancel{x^7}}}{\underset{2}{\cancel{8}}\cdot\underset{1}{\cancel{x^2}}} = \boxed{\frac{x^5}{2}}$

2.

$$\begin{array}{r|l}
 & 2x^2+3x-1 \\ \hline
3x^2 & 6x^4+9x^3-3x^2+4x+1 \\
 & \underline{-6x^4} \\
 & \quad +9x^3 \\
 & \quad \underline{-9x^3} \\
 & \qquad -3x^2 \\
 & \qquad \underline{-(-3x^2)} \\
 & \qquad\qquad 4x+1
\end{array}$$

(remainder. can't go further since degree of remainder is less than degree of divisor.)

$\frac{\overset{2}{\cancel{6}}\overset{x^2}{\cancel{x^4}}}{\underset{1}{\cancel{3}}\underset{1}{\cancel{x^2}}} = 2x^2 \qquad 2x^2(3x^2) = 6x^4$

$\frac{9x^3}{3x^2} = 3x \qquad 3x(3x^2) = 9x^3$

$$\frac{6x^4+9x^3-3x^2+4x+1}{3x^2} = \boxed{2x^2+3x-1+\frac{4x+1}{3x^2}}$$

3.

$$\begin{array}{r|l}
 & x^3+x^2-4x+5 \\ \hline
x+2 & x^4+3x^3-2x^2-3x+1 \\
 & \underline{-(x^4+2x^3)} \\
 & \quad x^3-2x^2 \\
 & \quad \underline{-(x^3+2x^2)} \\
 & \qquad -4x^2-3x \\
 & \qquad \underline{-(-4x^2-8x)} \\
 & \qquad\qquad 5x+1 \\
 & \qquad\qquad \underline{-(5x+10)} \\
 & \qquad\qquad\quad -9
\end{array}$$

$\frac{x^4}{x} = x^3 \qquad x^3(x+2) = x^4+2x^3$

$\frac{x^3}{x} = x^2 \qquad x^2(x+2) = x^3+2x^2$

$-\frac{4x^2}{x} = -4x \qquad -4x(x+2) = -4x^2-8x$

$\frac{5x}{x} = 5 \qquad 5(x+2) = 5x+10$

$$\frac{x^4+3x^3-2x^2-3x+1}{x+2} = \boxed{x^3+x^2-4x+5-\frac{9}{x+2}}$$

4. $$\frac{x^3-3x^2-4x+12}{x-3} \rightarrow \qquad \begin{array}{rl} & x^2 \quad - 4 \\ x-3 \, \big) & \overline{x^3-3x^2-4x+12} \\ & \underline{-(x^3-3x^2)} \; \downarrow \quad \downarrow \\ & \qquad\qquad -4x+12 \\ & \qquad\quad \underline{-(-4x+12)} \\ & \qquad\qquad\qquad 0 \end{array}$$

$$\frac{x^3}{x} = x^2$$

$$x^2(x-3) = x^3-3x^2$$

$$-\frac{4x}{x} = -4$$

$$-4(x-3)$$
$$= -4x+12$$

$$\frac{x^3-3x^2-4x+12}{x-3} = \boxed{x^2-4}$$

or . . . $x^3-3x^2-4x+12 = (x-3)(x^2-4)$

and $(x-3)$ is a <u>factor</u> of $x^3-3x^2-4x+12$.

5. $$\begin{array}{rl} & \qquad\qquad\quad 4x + 3 \\ 2x^2-x \, \big) & \overline{8x^3+2x^2+0x+1} \\ & \underline{-(8x^3-4x^2)} \; \downarrow \quad \downarrow \\ & \qquad 6x^2+0x+1 \\ & \quad \underline{-(6x^2-3x \qquad)} \\ & \qquad\qquad 3x+1 \end{array}$$

Note: the dividend is missing the "x" term, so a "0x" has been written in as a place holder.

$$\frac{8x^3}{2x^2} = 4x$$

$$4x(2x^2-x) = 8x^3-4x^2$$

$$\frac{6x^2}{2x^2} = 3$$

$$3(2x^2-x) =$$
$$6x^2-3x$$

we stop here since the highest power of the divisor is <u>more</u> than the highest power on the remainder.

$$\frac{8x^3+2x^2+1}{2x^2-x} = \boxed{4x+3+\frac{3x+1}{2x^2-x}}$$

6. $\dfrac{x^6-64}{x-2} \Rightarrow$

(many terms missing in divisor).

$$x-2\,\overline{)\,x^6+0x^5+0x^4+0x^3+0x^2+0x-64}\quad x^5+2x^4+4x^3+8x^2+16x+32$$

$$-(x^6-2x^5)$$
$$2x^5+0x^4$$
$$-(2x^5-4x^4)$$
$$4x^4+0x^3$$
$$-(4x^4-8x^3)$$
$$8x^3+0x^2$$
$$-(8x^3-16x^2)$$
$$16x^2+0x$$
$$(16x^2-32x)$$
$$32x-64$$
$$-(32x-64)$$
$$0$$

$$=\boxed{x^5+2x^4+4x^3+8x^2+16x+32}$$

or $x^6-64=(x-2)(x^5+2x^4+4x^3+8x^2+16x+32)$

7.

$$x-\tfrac{1}{2}\,\overline{)\,2x^4+x^3+0x^2+x-\tfrac{3}{4}}\quad 2x^3+2x^2+x+\tfrac{3}{2}$$

$$-(2x^4-x^3)$$
$$2x^3+0x^2$$
$$-(2x^3-x^2)$$
$$x^2+x$$
$$-(x^2-\tfrac{1}{2}x)$$
$$\tfrac{3}{2}x-\tfrac{3}{4}$$
$$-(\tfrac{3}{2}x-\tfrac{3}{4})$$
$$0$$

$\dfrac{2x^4}{x}=2x^3$

$2x^3(x-\tfrac{1}{2})=2x^4-x^3$

$\dfrac{2x^3}{x}=2x^2$ $\quad 2x^2(x-\tfrac{1}{2})$
$=2x^3-x^2$

$\dfrac{x^2}{x}=x$ $\quad x(x-\tfrac{1}{2})$
$=x^2-\tfrac{1}{2}x$

$\dfrac{\tfrac{3}{2}x}{x}=\tfrac{3}{2}$ $\quad \tfrac{3}{2}(x-\tfrac{1}{2})$
$=\tfrac{3}{2}x-\tfrac{3}{4}$

$$\frac{2x^4+x^3+x-\frac{3}{4}}{x-\frac{1}{2}}=\boxed{2x^3+2x^2+x+\frac{3}{2}}$$

8.

$$\begin{array}{r} 4x^3 + 13x^2 - 36x - 9 \\ x^2+9 \,\big)\, \overline{4x^5 + 13x^4 + 0x^3 + 108x^2 + 0x - 81} \\ -\underline{(4x^5 \qquad\quad + 36x^3)} \qquad\qquad\qquad\quad \\ 13x^4 - 36x^3 + 108x^2 \qquad\qquad \\ -\underline{(13x^4 \qquad\quad + 117x^2)} \qquad\qquad \\ -36x^3 - 9x^2 + 0x \qquad \\ -\underline{(-36x^3 \qquad\quad -324x)} \qquad \\ -9x^2 + 324x - 81 \\ -\underline{(-9x^2 \qquad\quad - 81)} \\ 324x \qquad \end{array}$$

$\frac{4x^5}{x^2} = 4x^3$

$4x^3(x^2+9) = 4x^5+36x^3$

$\frac{13x^4}{x^2} = 13x^2$

$13x^2(x^2+9) = 13x^4+117x^2$

$\frac{-36x^3}{x^2} = -36x$

$-36x(x^2+9) = -36x^3-324x$

$\frac{-9x^2}{x^2} = -9$ $\quad -9(x^2+9) = -9x^2-81$

$$\frac{4x^5+13x^4+108x^2-81}{x^2+9} = \boxed{4x^3+13x^2-36x-9+\frac{324x}{x^2+9}}$$

9.

$$\frac{5x^4+5x^2+5}{x^2-x+1} \Rightarrow$$

$$\begin{array}{r} 5x^2 + 5x + 5 \\ x^2-x+1 \,\big)\, \overline{5x^4 + 0x^3 + 5x^2 + 0x + 5} \\ -\underline{(5x^4 - 5x^3 + 5x^2)} \qquad\qquad \\ 5x^3 + 0x^2 + 0x \qquad \\ -\underline{(5x^3 - 5x^2 + 5x)} \qquad \\ 5x^2 - 5x + 5 \\ -\underline{(5x^2 - 5x + 5)} \\ 0 \end{array}$$

$$\frac{5x^4+5x^2+5}{x^2-x+1} = 5x^2+5x+5$$

note: the "5" could have been factored out of the divisor, making the problem

$$5\left[\frac{x^4+x^2+1}{x^2-x+1}\right] = 5\left[x^2+x+1\right] = 5x^2+5x+5$$

this would have been the resulting quotient.

Just don't forget to keep the multiple of 5 in your final answer.

10. $\dfrac{a^3 - b^3}{a-b}$

$$\begin{array}{r|l} & a^2 + ab + b^2 \\ \hline a-b & a^3 + 0a^2 + 0a - b^3 \\ & -(a^3 - a^2b) \\ \cline{2-2} & a^2b + 0a \\ & -(a^2b - ab^2) \\ \cline{2-2} & ab^2 - b^3 \\ & -(ab^2 - b^3) \\ \cline{2-2} & 0 \end{array}$$

(treat "a" as the variable when including missing terms.

$\frac{a^3}{a} = \boxed{a^2}$

$a^2(a-b) = a^3 - a^2b$

$\frac{a^2b}{a} = \boxed{ab}$

$ab(a-b) = a^2b - ab^2$

$\frac{ab^2}{a} = \boxed{b^2}$

$b^2(a-b) = ab^2 - b^3$

So, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

11. $\dfrac{x^4 + 3x^3 - 2x^2 - 3x + 1}{x+2}$

number that makes $(x+2) = 0$

coefficient of x^4 coefficient of x^3 coefficient of x^2 coefficient of x constant.

$$\begin{array}{r|rrrrr} -2 & 1 & 3 & -2 & -3 & 1 \\ & & -2 & -2 & 8 & -10 \\ \hline & 1 & 1 & -4 & 5 & \boxed{-9} \end{array}$$

add

multiply

remainder $\Rightarrow$

Quotient: $x^3 + x^2 - 4x + 5 - \frac{9}{x+2}$

coefficient of x^3 coefficient of x^2 coefficient of x constant.

quotient.

12. $\dfrac{x^6 - 64}{x-2}$

Note: In synthetic division, the powers of x are not written, so it is critical that any missing powers of x are represented with a "0" coefficient.

$$x^6 - 64 = x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 64$$

$$\begin{array}{r|rrrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & 0 & -64 \\ & \downarrow & 2 & 4 & 8 & 16 & 32 & 64 \\ \hline & 1 & 2 & 4 & 8 & 16 & 32 & \boxed{0} \end{array}$$

$$\frac{x^6 - 64}{x-2} = \boxed{x^5 + 2x^4 + 4x^3 + 8x^4 + 16x^5 + 32}$$

Additional Resources

Click on the links below to download a free online worksheet for more practice:

1. Go To https://cdn.kutasoftware.com/Worksheets/Alg1/Dividing%20Polynomials.pdf

2. For more help please contact the ***Math Assistance Area*.**