DIY: *Linear Inequalities in One Variable*

To review linear inequalities watch the following set of YouTube videos explaining what inequalities are and how to graph them followed by solving a system of linear inequalities. They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. Introduction to >, < , ≥ , ≤
2. Graphing Inequalities on a Number Line
3. Solving Multiple Step Inequalities
4. Interval Notation
5. Graphing Compound Inequalities

(Note: this presenter uses parentheses, ( ), instead of the open dot (○) on the graph to indicate that the endpoint is NOT included. He uses a bracket, [ ], instead of a closed dot (●) to indicate that the endpoint IS included. Many textbooks do this also, since it translates directly into the interval notation form.

6. Compound inequalities
7. Solving and Graphing a Compound Inequality, Ex. 1 ("and")
   Solving and Graphing a Compound Inequality, Ex. 2 ("and")
   Solving and Graphing a Compound Inequality, Ex. 2 ("or")

8. Absolute Value Equations
9. Solving Absolute Value Inequalities, example with >
10. Solving Absolute Value Inequalities, example with <
11. Absolute Value Inequalities, a special case

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Graph the following on a number line. Then write the solution in interval notation.
   a) $x < 10$ b) $x > 5$ c) $-5 > y$ d) $z \neq -101$ e) $a = 0$ f) $y \geq -9$ g) $x \leq 6$

2. Graph the following compound inequalities on a number line and write the solution in interval notation:
   a) $x < 1$ and $x > -10$ b) $x \geq 2$ or $x < 9$ c) $-4 < y \leq 7$ d) $y>0$ or $y \leq -3$ e) $-20 \leq z \leq -1$
   f) $p \geq \frac{1}{2}$ and $p < -5$ g) $-8 > t > 9$

3. Solve the inequality. Write your answer in interval notation.

   a) $2x < 10$ b) $3x+2 > 15$ c) $2b+2 \geq 29$ d) $-5c - 15 < 40c$
   e) $2z + 1 \neq -101$ f) $45 - 4a \leq 5$

4. Solve the following absolute value equations and inequalities. Express the solution in interval notation and graph on a number line.

a. $|7 - 3x| = 4$ b. $|2x + 9| - 19 \leq 0$ c. $3\,|52 - 5a| > 72$ d. $|3x - 34| < -5$

e. $12 - |6 - 3x| \geq 4$ f. $|2x - 4| = |5x + 1|$ g. $|12x + 81| \leq 0$ h. $|63 - 7x| \geq 27$

5. A machine part must have a diameter of 4.50 cm. A part is considered to be defective if it's diameter is off that measure by at least .05 cm. What diameter measurements can a part have to be considered NOT defective? Set up an absolute value inequality that can be solved to give the range of acceptable diameters, then solve.

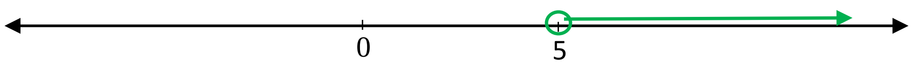
Answers:

1.

a)

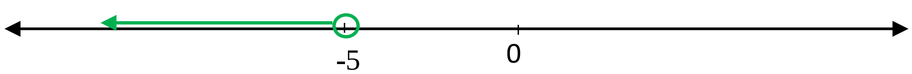
Interval: $(-\infty, 10)$

b)



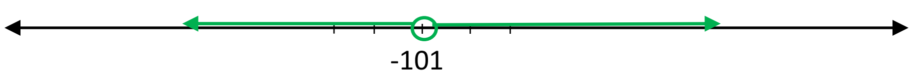
Interval: $(5, \infty)$

c)



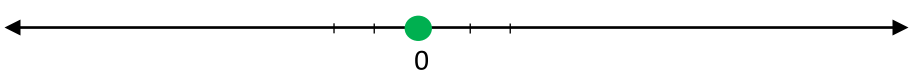
Interval: $(-\infty, -5)$

d)



Interval: $(-\infty, -101) \cup (-101, \infty)$

e)



Interval: $\{0\}$ (this isn't an interval, just one point))

f)

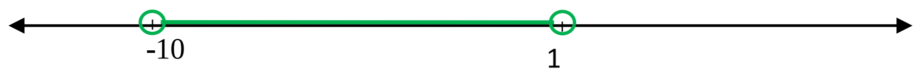
Interval: $[-9, \infty)$

g)

Interval: $(-\infty, 6]$

2.

a)

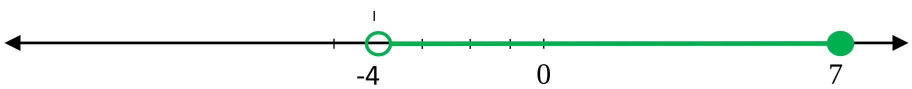


(-10,1)

b)

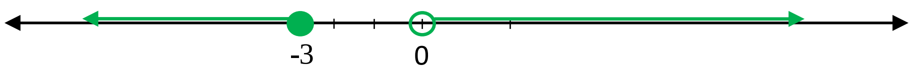
(-∞,∞)

c)



(-4,7]

d)



(-∞,-3] U (0, ∞)

e)

[-20 , -1]

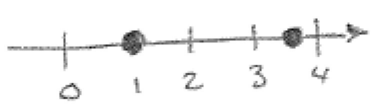
f) No solution, ∅

g) No solution, ∅

3.

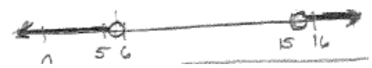
a)(-∞,5) b) $(4\frac{1}{3}, \infty)$ c) $[13\frac{1}{2}, \infty)$ d) $(-\frac{1}{3}, \infty)$ e) (-∞,-51) U (-51, ∞) f) [10, ∞)

4. a. $\left\{1, \frac{11}{3}\right\}$ Note: braces { } are used to indicate a list of single points, not an interval.



4. b. [-14, 5]

c. $\left(-\infty, \frac{28}{5}\right) \cup \left(\frac{76}{5}, \infty\right)$



d. no solution

e. $\left[-\frac{2}{3}, \frac{14}{3}\right]$

f. $\left\{-\frac{5}{3}, \frac{3}{7}\right\}$

g. $\left\{-\frac{27}{4}\right\}$

h. $\left(-\infty, \frac{36}{7}\right) \cup \left(\frac{90}{7}, \infty\right)$

5. Diameters which would not be defective are between 4.45 cm and 4.55 cm.

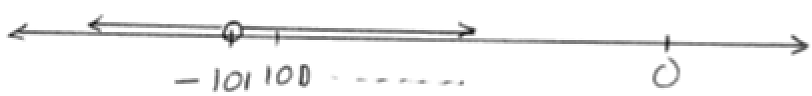
Detailed Solutions

1a) $x < 10$

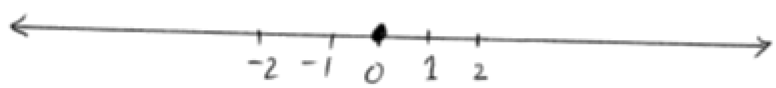
1b) $x > 5$

1c) $-5 > y$ or $y < -5$

1d) $z \neq -101$



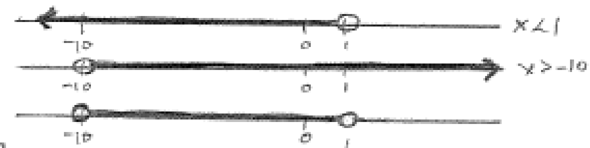
1e) $a = 0$



1f) $y \geq -9$

1g) $x \leq 6$

2\. a. $x<1$ and $x>-10$



(Could be written as $-10<x<1$)

Interval notation: $\boxed{(-10,1)}$

$x<1$ and $x>-10$: solution is interval where the two solutions overlap or intersect.

b. $x\geq 2$ or $x<9$

Solution is the union of the individual solutions to $x\geq 2$ and $x<9$.

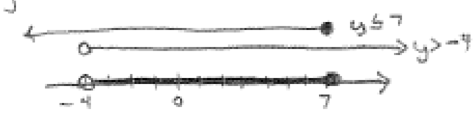
In this case, the solution is the entire number line.

interval: $\boxed{(-\infty,\infty)}$

c. $-4<y\leq 7$ can be written separately as

$-4<y$ and $y\leq 7$

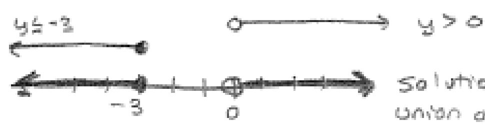
or $y>-4$ and $y\leq 7$



solution is where both inequalities are true.

interval: $\boxed{(-4,7]}$

d. $y>0$ or $y\leq -3$

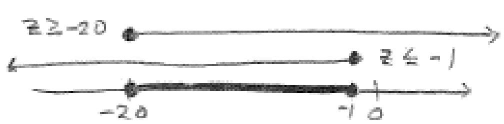


solution is the union of the solutions to the inequalities.

interval: $\boxed{(-\infty,-3]\cup(0,\infty)}$

e. $-20\leq z\leq -1$ which is the same as $-20\leq z$ and $z\leq -1$

or $z\geq -20$ and $z\leq -1$



Solution is the intersection of the two separate solutions

interval: $\boxed{[-20,-1]}$

f. $p\geq\frac{1}{2}$ and $p<-5$

no solution. Solns. do not intersect.

g. $-8>t>9$ means

$t<-8$ and $t>9$

no intersection of these intervals

so $\boxed{\text{no solution}}$

3. a. $2x < 10 \qquad \frac{2x}{2} < \frac{10}{2} \Rightarrow x < 5$

$\boxed{(-\infty, 5)}$

b. $3x + 2 > 15$

$\underline{\;-2 \qquad -2\;}$

$3x > 13 \rightarrow \frac{3x}{3} > \frac{13}{3} \rightarrow x > \frac{13}{3} = 4\frac{1}{3}$

$\boxed{(4\tfrac{1}{3}, \infty)}$

c. $2b + 2 \geq 29$

$\underline{\;-2 \qquad -2\;}$

$\frac{2b}{2} \geq \frac{27}{2} \rightarrow b \geq \frac{27}{2} = 13\frac{1}{2}$

$\boxed{[13\tfrac{1}{2}, \infty)}$

d. $-5c - 15 < 40c$

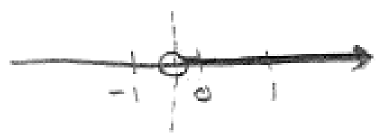
$\underline{-40c \qquad -40c}$

$-45c - 15 < 0$

$\underline{\;+15 \quad +15\;}$

$\frac{-45c}{-45} < \frac{15}{-45} \longrightarrow c > -\frac{1}{3}$

When dividing both sides by a negative number, the inequality sign reverses.



$\boxed{(-\tfrac{1}{3}, \infty)}$

e. $2z + 1 \neq -101$

$\underline{\;-1 \qquad -1\;}$

$\frac{2z}{2} \neq \frac{-102}{2} \rightarrow z \neq -51$

$\boxed{(-\infty, -51) \cup (-51, \infty)}$

3. f. $45 - 4a \leq 5$

$\underline{-45 \qquad\quad -45}$

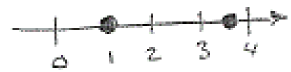
$\frac{-4a}{-4} \leq \frac{-40}{-4} \rightarrow a \geq 10$

sign reverses

$\boxed{[10, \infty)}$

4. a. $|7-3x| = 4$

soln: $\{1, 3\frac{2}{3}\}$

graph: 

$7-3x = 4$ or $7-3x = -4$

$\underline{-7 \qquad -7}$ $\qquad$ $\underline{-7 \qquad -7}$

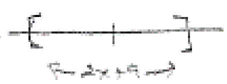
$-3x = -3$ $\qquad$ $\frac{-3x}{-3} = \frac{-11}{-3}$

$x = 1$ or $x = \frac{11}{3} = 3\frac{2}{3}$

b. $|2x+9| - 19 \leq 0$

$\underline{\qquad +19 \quad +19}$

$|2x+9| \leq 19 \Rightarrow -19 \leq 2x+9 \leq 19$ or $2x+9 \leq 19$ and $2x+9 \geq -19$



$-19 \leq 2x+9 \leq 19$

$\underline{-9 \qquad -9 \qquad -9}$

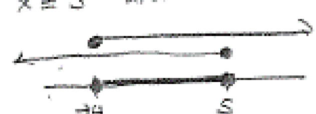
$\frac{-28}{2} \leq \frac{2x}{2} \leq \frac{10}{2}$

$-14 \leq x \leq 5$

$2x+9 \leq 19$ $\qquad$ $2x+9 \geq -19$

$2x \leq 10$ $\qquad$ $2x \geq -28$

$x \leq 5$ and $x \geq -14$



$\boxed{[-14, 5]}$

c. $\frac{3|52-5a|}{3} > \frac{72}{3} \Rightarrow |52-5a| > 24$

$52-5a > 24$ or $52-5a < -24$

$\underline{-52 \qquad -52}$ $\qquad$ $\underline{-52 \qquad -52}$

$\frac{-5a}{-5} > \frac{-28}{-5}$ $\qquad$ $\frac{-5a}{-5} < \frac{-76}{-5}$

reverses $\qquad$ reverses

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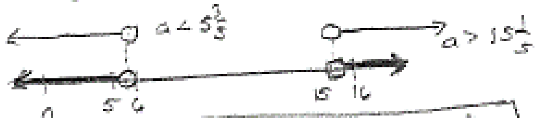
$a < \frac{28}{5} = 5\frac{3}{5}$ or $a > \frac{76}{5} = 15\frac{1}{5}$



$$\boxed{(-\infty, 5\tfrac{3}{5}) \cup (15\tfrac{1}{5}, \infty)}$$

d. $|3x - 34| < -5$ Since any absolute value is ≥ 0, this inequality has $\boxed{\text{no solution.}}$ or $\boxed{\emptyset}$

e. $12 - |6-3x| \geq 4$

$\quad -12 \qquad\qquad -12$

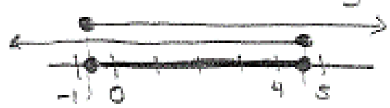
$$\frac{-|6-3x|}{-1} \geq \frac{-8}{-1} \quad \text{(reverses)} \Rightarrow |6-3x| \leq 8 \Rightarrow 6-3x \leq 8 \text{ and } 6-3x \geq -8$$

$6-3x \leq 8$ and $6-3x \geq -8$

$-6 \qquad -6 \qquad\qquad -6 \qquad -6$

$\frac{-3x}{-3} \leq \frac{2}{-3}$ reverse $\qquad \frac{-3x}{-3} \geq \frac{-14}{-3}$ reverse

$x \geq -\frac{2}{3} \qquad x \leq \frac{14}{3} = 4\frac{2}{3}$



$$\boxed{\left[-\tfrac{2}{3}, 4\tfrac{2}{3}\right]}$$

f. $|2x-4| = |5x+1| \Rightarrow 2x-4 = 5x+1$ or $2x-4 = -(5x+1)$

$2x - 4 = 5x + 1$

$-2x \qquad -2x$

$-4 = 3x + 1$

$-1 \qquad -1$

$\frac{-5}{3} = \frac{3x}{3}$

$-\frac{5}{3} = x$

$x = -\frac{5}{3} = -1\frac{2}{3}$

$2x-4 = -(5x+1)$

$2x - 4 = -5x - 1$

$+5x \qquad +5x$

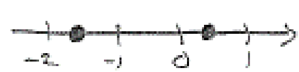
$7x - 4 = -1$

$+4 \qquad +4$

$\frac{7x}{7} = \frac{3}{7}$

$x = \frac{3}{7}$

soln: $\left\{-\frac{5}{3}, \frac{3}{7}\right\}$ graph:



4g. $|12x+81| \le 0$

$|12x+81| < 0$ or $|12x+81| = 0$

impossible.

Since any absolute value ≥ 0 the only possible solution would be when $12x+81=0$.

$12x = -81$

$x = -\frac{81}{12} = -\frac{27}{4} = -6\frac{3}{4}$

soln: $\left\{-\frac{27}{4}\right\}$ graph:

h. $|63-7x| \ge 27$

$63-7x \ge 27$ or $63-7x \le -27$

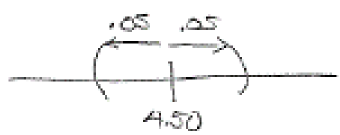
$-63 \quad -63$ $\quad$ $-63 \quad -63$

$\frac{-7x}{-7} \ge \frac{-36}{-7}$ reverse $\quad$ $\frac{-7x}{-7} \le \frac{-90}{-7}$ reverse

$x \le \frac{36}{7} = 5\frac{1}{7}$ $\quad$ $x \ge \frac{90}{7} = 12\frac{6}{7}$

$\boxed{\left(-\infty, 5\frac{1}{7}\right] \cup \left[12\frac{6}{7}, \infty\right)}$

i. diameter: 4.50 cm. ± .05 cm.



x = diameter of part

part is acceptable if: $\boxed{|x-4.50| < .05}$

(the difference between the diameter of the part and 4.50 cm. must be less than .05 cm.)

$-.05 < x - 4.50 < .05$

$+4.50 \quad +4.50 \quad +4.50$

$\boxed{4.45 < x < 4.55}$ or $\boxed{(4.45, 4.55) \text{ cm.}}$

Additional Resources

Click on the links below to download worksheets under "Inequalities":

1. Graphing one-variable inequalities
2. One-step inequalities
3. Two-step inequalities
4. Multi-step inequalities
5. Compound inequalities
6. Absolute value inequalities

Alternatively;

1. Go To http://www.kutasoftware.com/free.html
2. **Under "Inequalities":**
   - Graphing one-variable inequalities
   - One-step inequalities
   - Two-step inequalities
   - Multi-step inequalities

   - Compound inequalities
   - Absolute value inequalities
3. You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets

For help please contact the ***Math Assistance Area*****.**
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