DIY: Prime Factorization

Divisibility, Least Common Multiple (LCM), Greatest Common Factor (GCF)

To review more concepts involving prime factors, watch the following set of YouTube videos explaining prime factorization, finding the LCM (used when adding fractions) and the GCF (used when simplifying fractions). Following the videos are some practice problems for you to try, covering all the basic techniques, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. https://www.youtube.com/watch?v=FBbHzy7v2Kg Finding prime numbers
2. https://www.youtube.com/watch?v=tW97UU01ShY prime factorization, factor trees, exponent notation
3. https://www.youtube.com/watch?v=-RhdzNYfF-M finding prime factorization of a larger number
4. https://www.youtube.com/watch?v=Z6-LksV08qU finding the Least Common Multiple
5. https://www.youtube.com/watch?v=fRoWMakrIhw finding LCM using prime factors
6. https://www.youtube.com/watch?v=NtkjbVb3Zv8 finding the Greatest Common Factor
7. https://www.youtube.com/watch?v=qPVNMoCJ264 finding GCF of 3 numbers using exponents

(Note: see the next page for a summary table of common divisibility tests.)

These tests can be used to more quickly determine the factorization of a number, especially large numbers. This skill is used, for example, in simplifying fractions and radicals, and in adding/subtracting fractions.

Summary of Divisibility tests:

A number is divisible by…	If…	Example
2	The last digit is even (0, 2, 4, 6, or 8)	873 is not divisible by 2 (ends in odd digit). 960 ends in 0 so is divisible by 2.
3	The sum of the digits is divisible by 3	89748 is divisible by 3 since 8+9+4+4+8 = 36. Test can be repeated—for example, 3+6=9 which is divisible by 3.
4	The last 2 digits of the number form a number divisible by 4	37628 is divisible by 4 since the last two digits are 28 and 28 is divisible by 4. 4002 is not divisible by 4 since 02 (or 2) is not divisible by 4.
5	The last digit of the number is 5 or 0	8975 and 1060 are divisible by 5 but 5551 is not.
6	The number is divisible by both 2 and 3	548 is not divisible by 6. Although it ends with an even digit so is divisible by 2, the sum of the digits, 5+4+8=17 which is not divisible by 3.
7	When the last digit is removed, multiplied by 2, then subtracted from the remaining number, the result is divisible by 7. This test can be repeated.	Test 67935. Remove 5, multiply it by 2 (10). Subtract from remaining number: 6793 -10 = 6783. Repeat: 3x2= 6, then 678 – 6 = 672. Repeat: 2x2= 4, then 67-4= 63. 63 *is* divisible by 7, so 67934 is also. (In some cases, it may be simpler to just do long division.)
8	(Only practical for large numbers). The last 3 digits form a number divisible by 8.	9876543210. Is 210 divisible by 8? Use long division. 210 ÷ 8 = 26 with remainder 2, so 9876543210 is not divisible by 8. (We also know 210 is not divisible by 8 since it isn't divisible by 4!)
9	(Similar to test for 3). The sum of the digits is a number divisible by 9.	987654321. 9+8+7+6+5+4+3+2+1= 45. This is a multiple of 9 so 987654321 is divisible by 9. (Test can be repeated.)
10	The number ends with the digit 0.	6540 is divisible by 10. 6540 = 654x10
100	The number ends with the digits 00.	78200 = 782x100
1000	The number ends with 000	1234000 = 1234x1000

Practice problems: The following problems with answers use the techniques demonstrated in the above videos. Detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Use division to determine the following: a. Is 78 divisible by 2? b. Is 78 divisible by 3?

   c. Can 7423 be divided evenly by 3? d. Is 7423 divisible by 5? e. Is 7423 divisible by 7?

2. Use divisibility tests (other than using division) to determine the answers to the same questions in ex. 1

3. Find all the factors of 72. Hint: remember that factors occur in pairs. For example, 2 is a factor of 6 because 2 x 3 = 6. That means that 3 is also a factor of 6.

4. a. Find the prime factorization of 72. b. Find the prime factorization of 2600.

5. a. Find the LCM (least common multiple) of 25 and 30.

   b. Find the LCM of 18, 12, and 54

6. a. Find the GCF (greatest common factor) of 25 and 30 b. Find the GCF of 18, 12, and 54

7. Find both the LCM and the GCF for 24 and 35.

8. Find both the LCM and the GCF for 6 and 24.

Answers:

1.a. yes b. yes c. no d. no e. no 2. (same as question 1)

3. { 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72 }

4. a. $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \cdot 3^2$ b. $2600 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 13 = 2^3 \cdot 5^2 \cdot 13$

5. a. LCM of 25 and 30 = 150 b. LCM of 18, 12, and 54 = 108

6. a. GCF of 25 and 30 = 5 b. GCF of 18, 12, and 54 = 6

7. LCM = 840, GCF = 1 8. LCM = 24, GCF = 6

(See the next page for detailed solutions)

Detailed Solutions to Problems

1.a. Is 78 divisible by 2?

$$\begin{array}{r} 39 \\ 2\overline{)78} \\ \underline{6} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

yes. remainder = 0.
So, 2 and 39 are factors of 78.

b. Is 78 divisible by 3?

$$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

yes. remainder = 0.

We can say that 3 and 26 are factors of 78

c. Can 7423 be divided by 3?

$$\begin{array}{r} 2474 \\ 3\overline{)7423} \\ \underline{6} \\ 14 \\ \underline{12} \\ 22 \\ \underline{21} \\ 13 \\ \underline{12} \\ 1 \end{array}$$

no. remainder $\neq 0$.

3 is not a factor of 7423

d. Is 7423 divisible by 5?

$$\begin{array}{r} 1484 \\ 5\overline{)7423} \\ \underline{5} \\ 24 \\ \underline{20} \\ 42 \\ \underline{40} \\ 23 \\ \underline{20} \\ 3 \leftarrow \end{array}$$

no. remainder $\neq 0$
5 is not a factor of 7423

e. Is 7423 divisible by 7?

$$\begin{array}{r} 1060 \\ 7\overline{)7423} \\ \underline{7} \\ 42 \\ \underline{42} \\ 03 \\ \underline{0} \\ 3 \leftarrow \end{array}$$

no. remainder $\neq 0$.
7 is not a factor of 7423.

2. a. Is 78 divisible by 2? yes. if a number ends with an even digit, it can be divided by 2.

b. Is 78 divisible by 3? Add the digits of the number 7+8=15. If this number can be divided by 3, then the original number can be divided by 3. Still not sure? Repeat the test with the new number. 15 → 1+5=6 which is divisible by 3, so 78 is divisible by 3

c. Is 7423 divisible by 3? 7+4+2+3=16
then 1+6=7 which is not divisible by 3.
So 7423 is not divisible by 3.

d. Is 7423 divisible by 5? No. A number must end in the digit 0 or 5 to be divisible by 5.

e. Is 7423 divisible by 7?

The divisibility test for 7 is

1. remove the last digit and multiply it by 2.
2. subtract that from the remaining number (after that last digit was removed.)
3. If the resulting number is divisible by 7, then the original number is also. Not sure? The process can be repeated.

7243 → 724 → 3(2)=6

724 − 6 = 718

(not sure if 718 can be divided by 7, so repeat.)

718 → 71 → 8(2) = 16

71 − 16 = 55

55 is not divisible by 7, so 7243 is not divisible by 7.

3. Find all factors of 72. Hint: Remember that factors come in pairs.
Ex: 2 is a factor of 6 because $2 \times 3 = 6$. That also means 3 is a factor of 6.

$72 = 1 \times 72$ — 1, 72 are factors.
$72 = 2 \times 36$ — 2, 36 are factors.
$72 = 3 \times 24$ — 3, 24 are factors
$72 = 4 \times 18$ — 4, 18 are factors.
$72 = 5 \times$ — (no)
$72 = 6 \times 12$ — 6, 12 are factors
$72 = 7 \times$ — (no)
$72 = 8 \times 9$ — 8, 9 are factors.

since there are no factors of 72 between 8, 9, we have a complete list.

Factors of 72 = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}

4.a. Find the prime factorization of 72
Find any pair of numbers that multiply to 72 — 8×9 is one pair.

Both 8 and 9 are composite numbers, so we continue.

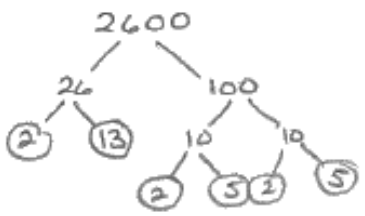
Circle the primes. Continue factoring with remaining composite number(s).

Done! All branches of the factor tree end in circled prime numbers.

$72 = 2 \times 2 \times 2 \times 3 \times 3$
$= 2^3 \cdot 3^2$ in exponent form.

4.b. Find the prime factorization for 2600.

note: any number ending in "0" is divisible by 10.
any number ending in "00" is divisible by 100.



$2600 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 13 = 2^3 \cdot 5^2 \cdot 13$

5. a. Find the LCM (least common multiple) of 25 and 30.

25	$5 \cdot 5$
30	$2 \cdot 3 \cdot 5$

$LCM = 2 \cdot 3 \cdot 5 \cdot 5 = 6 \cdot 25 = \boxed{150}$

another way!

multiples of 25:
25, 50, 75, 100, 125, 150, 175, 200

multiples of 30:
30, 60, 90, 120, 150, 180, 210

150 is the smallest number on both lists of multiples.

b. Find the LCM of 18, 12, and 54

18	$2 \quad 3 \cdot 3$
12	$2 \cdot 2 \; 3$
54	$2 \cdot \quad 3 \cdot 3 \cdot 3$

$LCM = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 4 \cdot 27 = \boxed{108}$

using exponents:

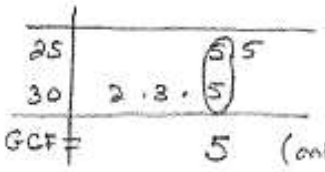
$18 = 2 \cdot 3^2$
$12 = 2^2 \cdot 3$
$54 = 2 \cdot 3^3$

For LCM, use highest power of each factor

$LCM = 2^2 \cdot 3^3 = 4 \cdot 27 = 108$

6.a. Find the GCF (greatest commas factor) of 25, 30

from ex. 5.a.



another way:

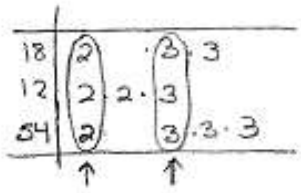
Factors of 25: 1, 5, 25

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

5 is largest number that appears on both lists of factors.

b. Find the GCF of 18, 12, and 54

from ex. 5b



another way:

Factors of 18: 1, 2, 3, 6, 9, 18

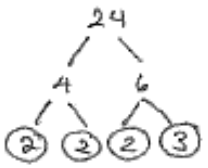
Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

6 is the largest number in all 3 lists of factors

7 Find both the LCM and GCF for 24 and 35

prime factorizations:



$24 = 2^3 \cdot 3$

$35 = 5 \cdot 7$

LCM:

	prime factors
24	$2 \cdot 2 \cdot 2 \cdot 3$
35	$5 \cdot 7$
LCM	$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 = 24(35) = \boxed{840}$

Since 24 and 35 have no prime factors in common, the least common multiple is the product of the two numbers.

GCF: again, 24 and 35 have no prime factors in common, so the only common factor is (1)

factors of 24: (1) 2, 3, 4, 6, 8, 12, 24

factors of 35: (1) 5, 7, 35

8. Find the LCM and GCF for 6 and 24.

LCM: since 24 is a multiple of 6, $\boxed{24}$ is the LCM

listing multiples: of 6: 6, 12, 18, (24), 30, 36, ...

of 24: (24), 48, ...

in table form:

24	(2) $2 \cdot 2 \cdot$ (3)
6	(2) (3)
LCM	$2 \cdot 2 \cdot 2 \cdot 3 = 24$
GCF	$2 \cdot 3 = 6$

GCF: again, since 6 is a factor of 24, it is the GCF of 6 and 24. factors of 24: 1, 2, 3, 4, (6), 8, 12, 24

factors of 6: 1, 2, 3, (6)

Additional Resources

1. Go To http://www.kutasoftware.com/freeipa.html
2. Find the "Number Theory" tile.
3. Select any skill in the tiles provided to practice (except "factoring monomials").
4. These are free online worksheets and once you answer them, scroll down to find the answers to the practice problems.
5. For further help please contact the ***Math Assistance Area***.