DIY: *Systems of Linear Equations*

To review solving systems of linear equations using non-matrix methods, watch the following set of YouTube videos. They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. **Systems of Linear Equations**
   a. Solving a system of equations by graphing
   b. Classifying a system of equations as Consistent, Inconsistent, Dependent & Independent Linear Systems
   c. Solving a system of equations by substitution Part 1
   d. Solving a system of equations by substitution Part 2
   e. Solving a system of equations by elimination Part 1
   f. Solving a system of equations by elimination Part 2
   g. Solving 3 equations in 3 variables using elimination
   h. Some applications of systems of linear equations

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Solve the following systems of equations by graphing:

a. y = 2x + 6
   y = -3x - 4

b. 3x + 2y = 4
   2x + 3y = 6

c. 6x + 3y = 21
   2x + y = 2

d. 4x + 5y = 15
   8x + 10 y = 30

e. $x = 2$
   $y = \frac{1}{2}$

f. y = 0
   y = x

2. Solve the following system of equations by substitution:

a. x = y+ 8
   x + y = 10

b. 12x + 3y = 21
   3x - 12y = 9

c. x -2y = 6
   2x - 4y = 12

d. 21y – 14x = 54
   -2x + 3y = 1

3. Solve the following system of equations by elimination

a. $5x - 4y = 21$
$10x + y = 7$

b. $2y - 7x = 6$
$8x - 5y = 4$

c. $y = 11x - 2$
$-22x = -2y - 4$

d. $x + y = 0$
$y = 2$

e. $-24x + 9y = 3$
$10y + 8x = 12$

f. $x = y$
$y = x + 4$

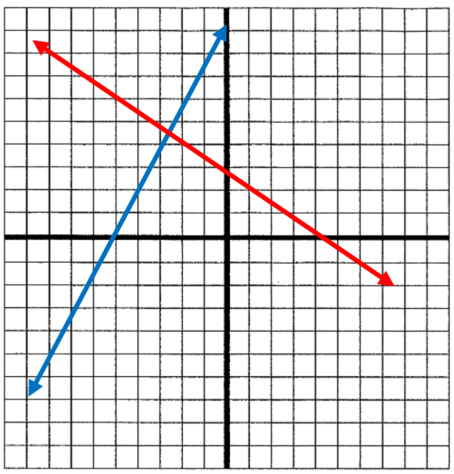
4. Classify the following lines as consistent, inconsistent, dependent and independent:

a. $2x + 4y = 10$
$x + 2y = 5$

b. $21y - 4x = 14$
$12x - 3y = 22$

c. $56x - 2y = 12$
$28x - y = 12$

d.



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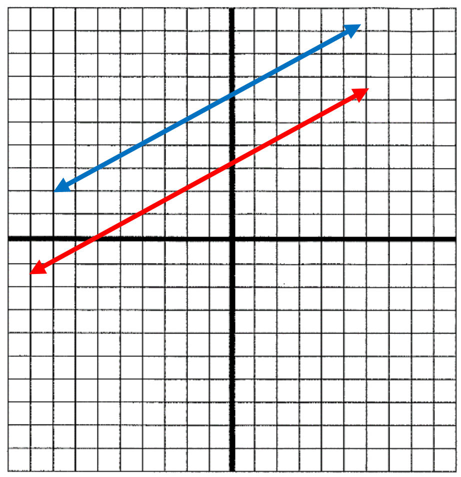

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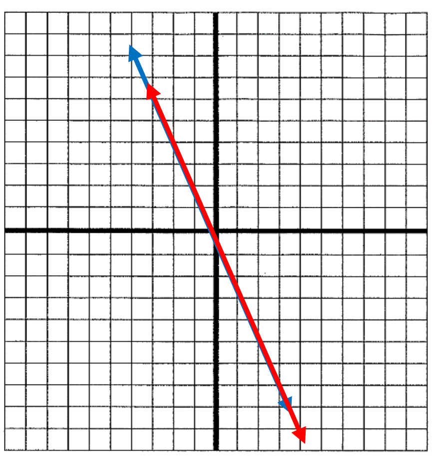
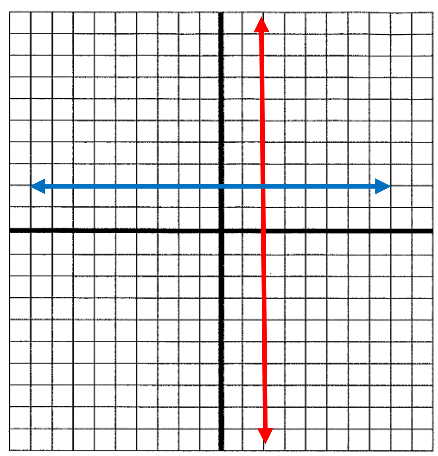


g.



5. Classify the following as parallel, perpendicular or neither:

a. x = y+ 11
x - y = 2

b. 2x + 3y = 24
8x - 12y = 24

c. x -2y = 6
24x + 12y = 60

d. 12y – 14x = 4
x + 10y = 121

6. Application problems:
   a. In 2016, city A had a population of 52,123 more than city B. Find the population of each city is the total population of the two cities is 150,895,023.
   b. The length of the top of a rectangular desk is 2.5 times its width. Find the dimensions of the desk if the perimeter is 35 ft.
   c. How many liters of a 10% alcohol solution and a 1 % solution should be added to obtain 60ℓ of a 4% solution?
   d. Maria bought two hotdogs and a drink in a ball park for $21.90 and Lizzy bought 3 hotdogs and 2 drinks for $35.35 Find the cost of a hotdog and a drink
   e. Two planes leave an airport in opposite directions from each other at the same time. Plane P is 100mph slower than Plane Q. Find the speed of each plane if they are 1000miles apart after 2 hours.

7. Solve the following system of equations using elimination:

$$x + 2y - z = 9$$
$$2x - y + 3z = -2$$
$$3x - 3y - 4z = 1$$

Answers:

1.
a) (-2,2) b) (0,2) c) No solution, Parallel lines d) Infinitely many solutions e) $(1, \frac{1}{2})$ f) (0,0)

2.
a) (9,1) b) $(\frac{31}{17}, \frac{-5}{17})$ c) Infinitely many solutions d) No solution, Parallel lines

3.
a) $(\frac{1}{5}$,-5) b) (-2,-4) c) Infinitely many solutions

d) (-2,2) e) $(\frac{1}{4}, 1)$ f) No solution, Parallel lines

4.
a) Consistent, dependent
b) Consistent, Independent
c) Inconsistent
d) Consistent , independent
e) Inconsistent
f) Consistent, dependent
g) Consistent , independent

5.
a) Parallel
b) Neither
c) Perpendicular
d) Neither

6.
a) Population Of City A = 75,473,573
Population Of City B = 75,421,450
b) Length = 12.5 ft.
Width = 5 ft.
c) 20ℓ of 10% solution should be added to 40 ℓ of 1% solution
d) Hot dog cost \$8.45 and Drink costs \$5
e) Speed of plan **P** = 200 mph
Speed of plan **Q** = 300 mph

7. (x, y, z) = (2, 3, -1)

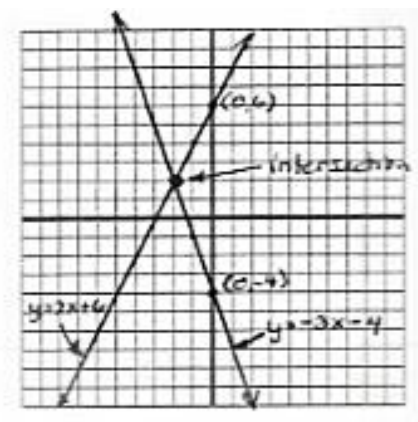
Detailed Solution for Solving Systems of Equations

1.a.

1.a. $y = 2x+6 \rightarrow$ slope = 2, y-int. (0,6)
$y = -3x-4 \rightarrow$ slope = -3, y-int (0,-4)

It appears that the lines intersect at (-2,2).

Checking by substituting into equations:
$y=2x+6 \rightarrow 2=2(-2)+6 \rightarrow 2=-4+6$ check!
$y=-3x-4 \rightarrow 2=-3(-2)-4 \rightarrow 2=6-4$ check!



1.b. $3x+2y=4$

$-3x \qquad -3x$

$\frac{2y}{2} = -\frac{3x}{2} + \frac{4}{2}$

$\boxed{y = -\frac{3}{2}x + 2}$

compare to $y = mx + b$

$m = -\frac{3}{2}$

y int $= (0,2)$

$2x + 3y = 6$

$-2x \qquad -2x$

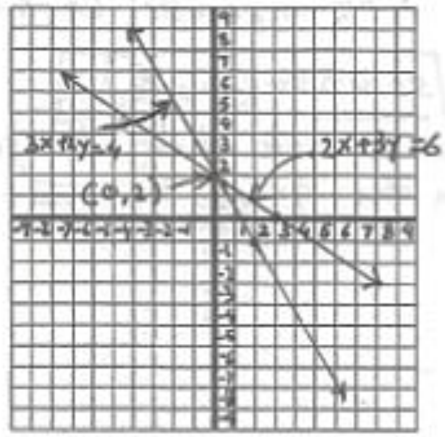
$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$

$\boxed{y = -\frac{2}{3}x + 2}$

$m = -\frac{2}{3}$

y int $= (0,2)$

$\boxed{\text{solution } (0,2)}$



1.c. $6x + 3y = 21$

$-6x \qquad -6x$

$\frac{3y}{3} = \frac{-6x}{3} + \frac{21}{3}$

$\boxed{y = -2x + 7}$

compare to $y = mx + b$

$m = -2$

y int $(0,7)$

$2x + y = 2$

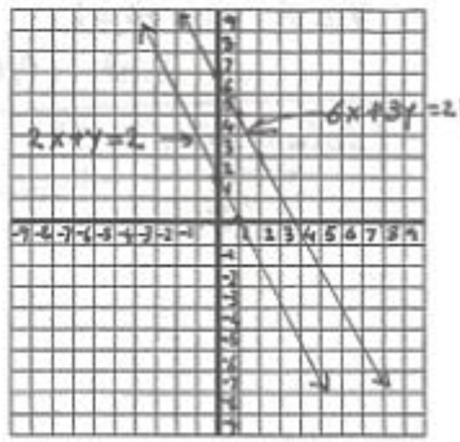
$-2x \qquad -2x$

$\boxed{y = -2x + 2}$

$m = -2$

y int $(0,2)$

$\boxed{\text{solution parallel lines no solution}}$



1.d. $4x+5y=15$

$-4x \qquad -4x$

$\frac{5y}{5} = \frac{-4x}{5} + \frac{15}{5}$

$\boxed{y = -\frac{4}{5}x + 3}$

compare to $y = mx + b$

$m = -\frac{4}{5}$

y int $(0,3)$

$8x + 10y = 30$

$-8x \qquad -8x$

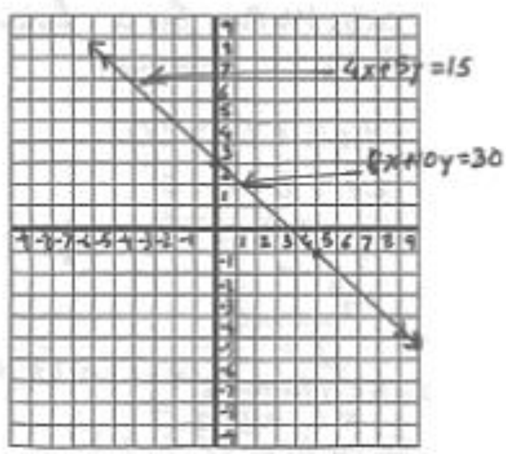
$\frac{10y}{10} = \frac{-8x}{10} + \frac{30}{10}$

$\boxed{y = \frac{-4}{5}x + 3}$

$m = -\frac{4}{5}$

y int $= (0,3)$

$\boxed{\text{solution: Infinitely many solutions}}$

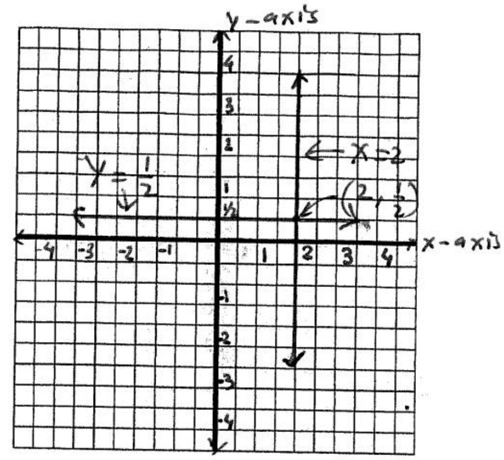


1.e.) $x = 2 \rightarrow$ parallel to y-axis

$y = \frac{1}{2} \rightarrow$ parallel to x-axis

$\boxed{\text{solution: } (2, \tfrac{1}{2})}$

*please note that the scale in the graph is 1 box $= \frac{1}{2}$ units

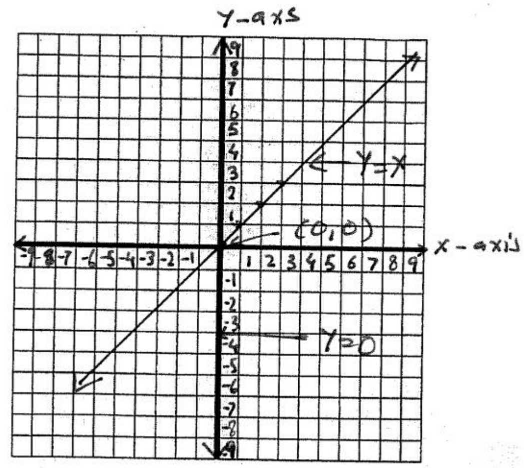


1.f. $y = 0 \rightarrow$ y-axis

$y = x \Rightarrow m = 1,$

Yint $(0,0)$

$\boxed{\text{solution: } (0,0)}$



2.a. $x = y + 8$ --- (1)

$x + y = 10$ --- (2)

substituting for x from (1) into 2 we get

$y + 8 + y = 10$

$2y + 8 = 10$

$\quad -8 \quad -8$

$\frac{2y}{2} = \frac{2}{2}$

$\boxed{y = 1}$

substituting $y = 1$ in (1) we get

$x = 1 + 8 = 9$

Solution: $\boxed{(9, 1)}$

check

$x = y + 8$

$9 = 1 + 8$

$9 = 9$ true

$x + y = 10$

$9 + 1 = 10$

$10 = 10$ true

The solution checks out.

2.b. $12x + 3y = 21$ --- ①
$3x - 12y = 9$ --- ②

solving ② for x

$3x - 12y = 9$
$+12y \quad +12y$

$\frac{3x}{3} = \frac{12y}{3} + \frac{9}{3}$

$\boxed{x = 4y + 3}$

substituting x in ① we get

$12(4y+3) + 3y = 21$

$48y + 36 + 3y = 21$

$51y + 36 = 21$
$-36 \quad -36$

$\frac{51y}{51} = \frac{-\cancel{15}5}{\cancel{51}17}$

$y = \frac{-5}{17}$ **

** $y = -\frac{5}{17}$

$x = 4y + 3$

$= 4\left(\frac{-5}{17}\right) + 3$

$= -\frac{20}{17} + 3$

$= \frac{-20+51}{17}$

$x = \frac{31}{17}$

Hence solution is $\boxed{\left(\frac{31}{17}, \frac{-5}{17}\right)}$

check

① $\rightarrow 12\left(\frac{31}{17}\right) + 3\left(\frac{-5}{17}\right) = 21$

$\frac{372 - 15}{17} = 21$

$\frac{\cancel{357}21}{\cancel{17}} = 21$; $21 = 21$ true

② $\rightarrow 3\left(\frac{31}{17}\right) - 12\left(\frac{-5}{17}\right) = 9$

$\frac{93}{17} + \frac{60}{17} = 9$

$\frac{\cancel{153}9}{\cancel{17}} = 9$; $9 = 9$ true

2.c. $x - 2y = 6$ --- ①
$2x - 4y = 12$ --- ②

solving ① for x we get

$x - 2y = 6$
$+2y \quad +2y$

$\boxed{x = 2y + 6}$

substituting x in ② we get

$2(2y+6) - 4y = 12$

$4y + 12 - 4y = 12$

$12 = 12$

Always true
Thus there are infinitely many solutions

2.d. $21y - 14x = 54$ --- ①
$-2x + 3y = 1$ --- ②

solving ② for x we get

$-2x + 3y = 1$
$-3y \quad -3y$

$\frac{-2x}{-2} = \frac{1}{-2} - \frac{3y}{-2}$

$x = +\frac{3}{2}y - \frac{1}{2}$

Substituting for x in ① we get

$21y - 14\left(\frac{3}{2}y - \frac{1}{2}\right) = 54$

$21y - 21y + 7 = 54$

$7 = 54$

not true
Hence no solutions, parallel lines

3.a. $5x - 4y = 21$ -- ①
$10x - y = 7$ -- ②

Multiplying ① by -2 we get

$-10x + 8y = -42$
$10x - y = 7$

$7y = -35$ Adding the two equations

$y = -35/7$

$\boxed{y = -5}$

Substituting y in ① we get

$5x - 4(-5) = 21$
$5x + 20 = 21$
$-20 \quad -20$
$5x = 1$
$\boxed{x = \frac{1}{5}}$

$\boxed{\text{Solution is } (\frac{1}{5}, -5)}$

check

plugging $x = \frac{1}{5}$ and $y = -5$ in ① we get

$5(\frac{1}{5}) - 4(-5) = 21$
$1 + 20 = 21$
$21 = 21$ TRUE

plugging $x = \frac{1}{5}$ and $y = -5$ in ② we get

$10^2(\frac{1}{5}) - (-5) = 7$
$2 + 5 = 7$
$7 = 7$ TRUE

3.b. $2y - 7x = 6 \rightarrow -7x + 2y = 6$ ①
$8x - 5y = 4 \rightarrow 8x - 5y = 4$ ②

Multiplying ① by 5 and ② by 2 we get

$-35x + 10y = 30$
$16x - 10y = 8$

$-19x = 38$

$\frac{-19x}{-19} = \frac{38}{-19}$

$\boxed{x = -2}$

Substituting $x = -2$ in ① we get

$2y - 7(-2) = 6$
$2y + 14 = 6$
$-14 \quad -14$
$2y = -8$
$\boxed{y = -4}$

$\boxed{\text{solution is } (-2, -4)}$

check

Plugging $x = -2$ and $y = -4$ in ① we get

$2(-4) - 7(-2) = 6$
$-8 + 14 = 6$
$\boxed{6 = 6}$ TRUE

Plugging $x = -2$ and $y = -4$ in ② we get

$8(-2) - 5(-4) = 4$
$-16 + 20 = 4$
$\boxed{4 = 4}$ TRUE

3.c. $Y = 11x - 2$ --- (1)

$-22x = -2y - 4$ --- (2)

Rearranging (1) and (2) we get

$-11x + Y = -2$ --- (1)

$-22X + 2Y = -4$ --- (2)

Multiplying (1) by -2 we get

$$22x - 2y = +4$$
$$-22X + 2Y = -4$$
$$0 = 0$$

Always true hence they have infinitely many solution

3.d. $x + y = 0$ --- (1)

$Y = 2$ --- (2)

Multiplying (2) by -1 and we get

$$x + \cancel{y} = 0 \quad \text{--- (2)}$$
$$-\cancel{Y} = -2 \quad \text{--- (1)}$$
$$x = -2$$

substituting it in (1) we get

$-2 + y = 0$

$+2 \quad +2$

$Y = 2$

$\boxed{\text{solution } (-2, 2)}$

check

$-2 + 2 = 0$ and $Y = 2$

$0 = 0$ $\quad 2 = 2$

True $\quad$ True

the solution checks out

3.e. $-24x + 9y = 3$ --- (1)

$10y + 8X = 12$ --- (2)

Multiply (2) by 3 and rearranging we get

$$\cancel{24X} + 30Y = 36 \quad \text{--- (2)}$$
$$-\cancel{24X} + 9Y = 3 \quad \text{--- (1)}$$
$$\frac{39Y}{39} = \frac{39}{39}$$
$$Y = 1$$

Substituting $y = 1$ in (1) we get

$-24x + 9(1) = 3$

$-24x + 9 = 3$

$-9 \quad -9$

$\frac{-24x}{-24} = \frac{-6}{-24}$

$\boxed{x = \frac{1}{4}}$

$\boxed{\text{solution is } (\frac{1}{4}, 1)}$

check

Plugging $x = \frac{1}{4}$ & $y = 1$ in (1) we get

$-24(\frac{1}{4}) + 9(1) = 3$

$-6 + 9 = 3$

$\boxed{3 = 3}$ TRUE

Plugging $x = \frac{1}{4}$ and $y = 1$ in (2) we get

$10(1) + 8(\frac{1}{4}) = 12$

$10 + 2 = 12$

$\boxed{12 = 12}$

TRUE.

3.f. 1. $x = y$ --- (1)

$y = x + 4$ --- (2)

Rearranging we get

$\cancel{x} - \cancel{y} = 0$ --- (1)

$-\cancel{x} + \cancel{y} = 4$ --- (2)

$0 = 4$

False statement

Hence the lines are parallel and have no solution

4.a. $2x + 4y = 10$ --- (1)

$x + 2y = 5$ --- (2)

we can observe that if we multiply (2) by 2 we get

$2x + 4y = 10$

which is same as (1) and hence there the two lines are the same

Hence

$\boxed{\text{Consistent and dependent}}$

Note: If one equation is a multiple of the other then the lines are consistent and dependent

These lines have infinitely many solution

4.b. $21y - 4x = 14$ --- (1)

$12x - 3y = 22$ --- (2)

Rearranging the equations to $y = mx + b$ we get

$21y - 4x = 14$

$+4x \qquad +4x$

$\frac{21y}{21} = \frac{4x}{21} + \frac{\cancel{14}\,2}{\cancel{21}\,3}$

$\boxed{y = \frac{4}{21}x + \frac{2}{3}}$ - (1)

$12x - 3y = 22$

$-12x \qquad -12x$

$\frac{-3y}{-3} = \frac{-12x}{-3} + \frac{22}{-3}$

$\boxed{y = 4x - \frac{22}{3}}$ - (2)

The two lines have different slopes hence they are $\boxed{\text{consistent and independent}}$

The lines intersect and have one solution

4.c. $56x - 2y = 12$ --- (1)

$28x - y = 12$ --- (2)

Rearranging (1) and (2) such that we get the lines in the standard form $y = mx + b$

$$56x - 2y = 12$$
$$-56x \qquad -56x$$
$$\frac{-2y}{-2} = \frac{-\overset{28}{\cancel{56}}x}{-2} + \frac{12}{-2}$$
$$\boxed{y = 28x - 6} \text{ --- (1)}$$

$$28x - y = 12$$
$$-28x \qquad -28x$$
$$\frac{-y}{-1} = \frac{-28x}{-1} + \frac{12}{-1}$$
$$\boxed{y = 28x - 12} \text{ --- (2)}$$

Both lines (1) and (2) have the same slope but have different y-intercepts. Hence the lines are parallel and have no solution. We can conclude that the equations are

$\boxed{\text{inconsistent}}$

4.d. The lines intersect and have one solution. Hence the system of equations is consistent and independent

4.e.) The two lines are parallel and have no solution Hence the system of equations is inconsistent

4.f.) The two lines overlap and have infinitely many solutions. Hence the system is consistent and dependent

4.g. The two lines intersect and have one solution. Hence the system is consistent and independent

5.a.) $x = y + 11$ --- (1)

$x - y = 2$ --- (2)

Rearranging (1) and (2) we get

$y = x - 11$ --- (1)

$y = x - 2$ --- (2)

Slope of (1) $m_1 = 1$

and slope of (2) $m_2 = 1$

So both the lines have same slope.

Hence they are parallel

5.b. $2x + 3y = 24$ --- (1)

$8x - 12y = 24$ --- (2)

Rearranging (1) and (2) we get

$2x + 3y = 24$

$-2x \quad -2x$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{24}{3}$

$y = -\frac{2}{3}x + 8$ --- (1)

$8x - 12y = 24$

$-8x \quad -8x$

$\frac{-12y}{-12} = \frac{-8x}{-12} + \frac{24}{-12}$

$y = \frac{2}{3}x - 2$ --- (2)

Slope $m_1 = \frac{-2}{3}$, $m_2 = \frac{2}{3}$

the slopes are neither same or opposite reciprocals

hence the lines are neither parallel nor perpendicular

5.c. $x - 2y = 6$ --- (1)

$24x + 12y = 60$ --- (2)

Rearranging we get

$x - 2y = 6$

$-x \quad -x$

$\frac{-2y}{-2} = \frac{-x}{-2} + \frac{6}{-2}$

$y = \frac{1}{2}x - 3$ --- (1)

$24x + 12y = 60$

$-24x \quad -24x$

$\frac{12y}{12} = -\frac{24}{12}x + \frac{60}{12}$

$y = -2x + 5$ --- (2)

slope $m_1 = \frac{1}{2}$, $m_2 = -2$

The slopes are opposite reciprocals of each other.

Hence the lines are perpendicular

5.d. $12y - 14x = 4$ --- (1)

$x - 10y = 121$ --- (2)

Rearranging we get

$12y - 14x = 4$

$+14x \quad +14x$

$\frac{12y}{12} = \frac{14}{12}x + \frac{4}{12}$

$y = \frac{7}{6}x + \frac{1}{3}$ --- (1)

$x - 10y = 121$

$-x \quad -x$

$\frac{-10y}{-10} = \frac{-x}{-10} + \frac{121}{-10}$

$y = \frac{1}{10}x - \frac{121}{10}$ --- (2)

slopes $m_1 = \frac{7}{6}$

$m_2 = \frac{1}{10}$

The slope are different hence neither

16.a. Population of city A = x

Population of city B = y

Hence $x = y + 52{,}123$

or by rearranging $x - y = 52{,}123$ ---- (1)

$x + y = 150{,}895{,}023$ ---- (2)

This is a system of two linear equations. We can choose any method of solving them. Let us use elimination.

$$x - \cancel{y} = 52{,}123$$
$$x + \cancel{y} = 150{,}895{,}023$$

$$\frac{2x}{2} = \frac{150{,}947{,}146}{2}$$

$$\boxed{x = 75{,}473{,}573}$$

$$\begin{array}{r} 75473573 \\ 2\overline{)150947146} \\ -14 \\ \hline 10 \\ -10 \\ \hline 09 \\ -8 \\ \hline 014 \\ -14 \\ \hline 07 \\ -6 \\ \hline 11 \\ -10 \\ \hline 014 \\ -14 \\ \hline 06 \\ -6 \\ \hline 0 \end{array}$$

substituting $x = 75{,}473{,}573$ in (1) we get

$$75{,}473{,}573 - y = 52{,}123$$
$$-75473573 \qquad -75{,}473{,}573$$

$$\frac{-y}{-1} = \frac{-75{,}421{,}450}{-1}$$

$$\boxed{y = 75{,}421{,}450}$$

Population of city A = 75,473,573
and Population of city B = 75,421,450

6.b.

let length of the desk = l

width of the desk = w

$$\boxed{l = 2.5w} \quad \text{--- (1)}$$

perimeter = $2(l+w)$

$$\boxed{35 = 2(l+w)} \quad \text{--- (2)}$$

We have a system of linear equations we can choose any method to solve. Let us use substitution.

Substituting l from (1) into (2) we get

$$35 = 2(2.5w + w)$$

$$35 = 2(3.5w)$$

$$\frac{35}{7} = \frac{7w}{7}$$

$$5 = w \quad \text{or} \quad \boxed{w = 5ft}$$

substituting w in (1) we get

$$\boxed{l = (2.5)(5) = 12.5ft}$$

$\boxed{\text{The length of the desk is 12.5ft and the width is 5ft}}$

6.c.

	% alcohol	Liters of solution	Liters of Pure alcohol
Solution 1	$10\% = 0.1$	x	$0.1x$
Solution 2	$1\% = 0.01$	y	$0.01.y$
Mixture	$4\% = 0.04$	$60L$	$(0.04)(60) = 2.4L$

$$\boxed{X + Y = 60} \quad \text{---} (1)$$

$$\boxed{0.1X + 0.01Y = 2.4} \quad \text{--} (2)$$

we have a system of linear equation we can choose any method to solve. Let us use elimination

Multiplying (2) by -100 and (1) by 2 we get

$$
\begin{array}{r}
-10x - \cancel{Y} = -240 \\
x + \cancel{Y} = 60 \\
\hline
-9x = -180
\end{array}
$$

$$\frac{-9x}{-9} = \frac{-180}{-9}$$

$$\boxed{X = 20\ell}$$

Substituting $x = 20$ in (1) we get

$$20 + Y = 60$$
$$-20 \qquad -20$$

$$\boxed{\therefore Y = 40\ell}$$

We need to mix 20ℓ of 10% solution and 40ℓ of 1% to get the desired solution

6.d.) Let the price of one hotdog $= \$x$

and the price of one drink $= \$y$

For Maria

$$2X + Y = 21.90 \quad \text{---} (1)$$
$$3X + 2Y = 35.35 \quad \text{---} (2)$$

We now have a system of linear equations We can choose any method to solve. Let us use elimination.

Multiplying ① by -2 we get

$$-4x - \cancel{2y} = -43.8$$
$$3x + \cancel{2y} = 35.35$$

$$\frac{-x}{-1} = \frac{-8.45}{-1}$$

$$\boxed{x = \$8.45}$$

substituting $x = 8.45$ in ② we get

$$3(8.45) + 2y = 35.35$$

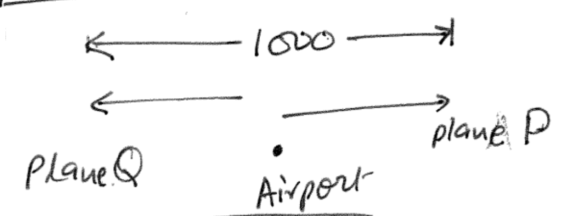
$$25.35 + 2y = 35.35$$
$$-25.35 \qquad\qquad -25.35$$

$$\frac{2y}{2} = \frac{10}{2}$$

$$\boxed{y = 5}$$

Cost of a hotdog is \$8.45 and a drink is \$5

6.e.



*note speed = $\frac{\text{Distance}}{\text{time}}$

Let speed of plane P = x mph and speed of plane Q = y mph

Distance travelled by:

plane P in 2 hours = $2x$

plane Q in 2 hours = $2y$

Hence $2x + 2y = 1000$ --- ①

Also $x = y - 100$ --- ②

Now we have a system of equations. Let us use substitution. substituting x from ② in ① we get

$$2(y-100) + 2y = 1000$$
$$2y - 200 + 2y = 1000$$
$$4y - 200 = 1000$$

$$4y - 200 = 1000$$
$$+200 \quad\quad +200$$

$$\frac{4y}{4} = \frac{\cancel{1200}\ 300}{\cancel{4}\ 1}$$

$$\boxed{y = 300 \text{ mph}}$$

substituting $y = 300$ in ② we get

$$x = 300 - 100$$

$$\boxed{x = 200 \text{ mph}}$$

$\boxed{\text{Speed of plane P is 200mph and speed of plane Q is 300 mph.}}$

7. $x + 2y - z = 9$ ①
$2x - y + 3z = -2$ ②
$3x - 3y - 4z = 1$ ③

*Any of the variables could be eliminated first, but we will eliminate x first.

Step 1: eliminate one variable* through forming combinations of the equations so that the system is reduced to 2 equations in 2 variables.

multiply ① by (-2): add ②

$$\begin{array}{r} -2x - 4y + 2z = -18 \\ 2x - y + 3z = -2 \\ \hline -5y + 5z = -20 \end{array}$$

to simplify, divide both sides by (-5): $y - z = 4$ ④

now combine ③ with one of the other equations to also eliminate x:

multiply ① by (-3): add ③

$$\begin{array}{r} -3x - 6y + 3z = -27 \\ 3x - 3y - 4z = 1 \\ \hline -9y - z = -26 \end{array}$$ ⑤

Step 2: Solve the 2 equation system by either elimination or substitution:

$y - z = 4$ ④
$-9y - z = -26$ ⑤

switching to substitution, eqn. ④ is solved for y: $y = z + 4$

then substitute $z+4$ for y in ⑤

$$-9(z+4) - z = -26$$
$$\begin{array}{r} -9z - 36 - z = -26 \\ +36 \quad\quad +36 \\ \hline -10z = 10 \end{array}$$
$$\boxed{z = -1}$$

substituting back to find y: $y = z + 4$
$= -1 + 4$
$\boxed{y = 3}$

then substitute y and z values back into one of original three equations:

① $x + 2(3) - (-1) = 9$
$x + 6 + 1 = 9$
$x + 7 = 9$
$\boxed{x = 2}$

Solution: $\boxed{(x,y,z) = (2,3,-1)}$ **

** checking in other equations:

② $2(2) - 3 + 3(-1) = -2$
$4 - 3 - 3 = -2$
$-2 = -2$ ✓

③ $3(2) - 3(3) - 4(-1) = 1$
$6 - 9 + 4 = 1$
$10 - 9 = 1$
$1 = 1$ ✓

Note: Solving systems of three equations in three variables is more efficiently handled using a matrix method such as Gauss-Jordan elimination.

Additional Resources

1. Go To http://www.kutasoftware.com/free.html

2. **Under "Systems of Equations and Inequalities":**
   - Solving systems of equations by graphing
   - Solving systems of equations by substitution
   - Solving systems of equations by elimination
   - Systems of equations word problems

3. You can print out the workSheets and work on them. The solutions are provided at the end of the worksheets

4. For help, please contact the ***Math Assistance Area*.**