

DIY: Simplifying Exponential Expressions

To review Exponents and Logarithms, watch the following set of YouTube videos explaining the basic techniques and rules, followed by some practice problems for you to try covering all the basic techniques, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. <https://www.youtube.com/watch?v=Kr16rdBMX4o> (basic exponent properties)
2. <https://www.youtube.com/watch?v=wsaH5CARIHI> (examples of applying exponent rules, part 1)
3. <https://www.youtube.com/watch?v=X72qoK6i2B8> (examples of applying exponent rules, part 2)
4. <https://www.youtube.com/watch?v=m4sX4FQOyoo> (negative and fractional exponents)
5. <https://www.youtube.com/watch?v=gsYEE-jxS5k> (examples of applying exponent rules, part 3)
6. <https://www.youtube.com/watch?v=bi23910GN60> (simplifying expressions with negative exponents)
7. <https://www.youtube.com/watch?v=7gTfmXhFOVo> (examples with negative exponents, part 2)
8. <https://www.youtube.com/watch?v=c4aiYf3fzVQ> (examples with negative exponents, part 3)
9. <https://www.youtube.com/watch?v=wp2Ho985DyI> (rational exponents, examples)
10. <https://www.youtube.com/watch?v=odnQGV0VAKg> (radical notation and simplifying radicals)

Remember the basic rules:

$$\begin{array}{lllll} 1. x^a * x^b = x^{a+b} & 2. \frac{x^a}{x^b} = x^{a-b} & 3. (x^a)^b = x^{ab} & 4. x^0 = 1 \ (x \neq 0) & 5. x^{-a} = \frac{1}{x^a} \\ 6. \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} & 7. (xy)^a = x^a y^a & 8. \left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a & & \end{array}$$

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are given after the answer section. For further assistance and help please contact [Math Assistance Area](#).

$$1. \text{ a. Simplify: } y^3 * y^8 * y^2 = \underline{\hspace{2cm}} \quad 1. \text{ b. Fill in the blank: } \frac{t^4 * t^4}{t^2} = t^{\square} \quad 1. \text{ c. Simplify: } \frac{x^3}{x^6} = \underline{\hspace{2cm}}$$

$$2. \text{ a. } (a^3 b^5)^4 = \underline{\hspace{2cm}} \quad 2. \text{ b. } \left(\frac{3}{5x}\right)^{-2} = \underline{\hspace{2cm}} \quad 3. (-2xy^2z^3)(-3xy^4z^3) = \underline{\hspace{2cm}}$$

4. $\left(\frac{4ab^3}{3a^2b^6}\right)^{-3} = \underline{\hspace{2cm}}$ 5. a. $5^2 = \underline{\hspace{1cm}}$ b. $-5^2 = \underline{\hspace{1cm}}$ c. $(-5)^2 = \underline{\hspace{1cm}}$ d. $-5^{-2} = \underline{\hspace{1cm}}$

6. $\frac{2^0}{3^0+4^0} = \underline{\hspace{2cm}}$ 7. $\frac{(2x^{-3}y^4)^{-2}}{(3x^{-6}y^{-2})^3} = \underline{\hspace{2cm}}$ 8. $4^{-1} + 5^{-1} = \underline{\hspace{2cm}}$

9. $(x^2 - y^2)^2 = \underline{\hspace{2cm}}$ 10. $\frac{0^{10}}{10^0} = \underline{\hspace{2cm}}$ 11. $(4x^{-2}y)^{-2}(2xy^{-3})^3$

12. $\frac{(3a^{-2}b^3)^2}{(a^3b^{-2})^{-3}} * (5a^{-1}b^{-4})^{-1}$

Answers:

1. a. y^{13} 1.b. t^6 1.c. $\frac{1}{x^3}$ 2.a. $a^{12}b^{20}$ 2.b. $\frac{25x^2}{9}$ 3. $6x^2y^6z^6$

4. $\frac{27a^3b^9}{64}$ 5.a. 25 b. -25 c. 25 d. $\frac{-1}{25}$ 6. $\frac{1}{2}$ 7. $\frac{x^{24}}{108y^2}$ 8. $\frac{9}{20}$

9. $x^4 - 2x^2y^2 + y^4$ 10. 0 11. $\frac{x^7}{2y^{11}}$ 12. $\frac{9a^6b^4}{5}$

Detailed Solutions to Exponent Simplification Problems

1. a. $y^3 \cdot y^8 \cdot y^2 = y^{3+8+2} = \boxed{y^{13}}$ Visually, $y^3 \cdot y^8 \cdot y^2 = \underbrace{y \cdot y \cdot y}_{y^3} \cdot \underbrace{y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}_{y^8} \cdot \underbrace{y \cdot y}_{y^2}$ (11 y's multiplied)

b. $\frac{t^4 \cdot t^4}{t^2} = \frac{t^{4+4}}{t^2} = \frac{t^8}{t^2} = t^{8-2} = \boxed{t^6}$

visually: $\frac{t^4 \cdot t^4}{t^2} = \frac{\cancel{t} \cdot \cancel{t} \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t}{\cancel{t} \cdot \cancel{t}} = t^6$

c. $\frac{x^3}{x^6} = x^{3-6} = x^{-3} = \frac{1}{x^3}$ Visually: $= \frac{x \cdot x \cdot x \cdot 1}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x} = \boxed{\frac{1}{x^3}}$

or $= \frac{x^3}{x^6} = \frac{1}{x^{6-3}} = \frac{1}{x^3}$ (subtract smaller exponent from larger exponent and put result on the side of the fraction where the larger exponent was.)

2. a. $(a^3 b^5)^4 = (a^3)^4 \cdot (b^5)^4 = \boxed{a^{12} b^{20}}$

b. $\left(\frac{3}{5x}\right)^{-2} = \left(\frac{5x}{3}\right)^2 = \frac{5^2 x^2}{3^2} = \boxed{\frac{25x^2}{9}}$

3. $(-2xy^2z^3)(-3xy^4z^3) = (-2)(-3)(x)(x)(y^2)(y^4)(z^3)(z^3)$
 $= 6x^2y^{2+4}z^{3+3}$
 $= \boxed{6x^2y^6z^6}$

4. $\left(\frac{4ab^3}{3a^2b^6}\right)^{-3} = \left(\frac{3a^2b^6}{4ab^3}\right)^3 = \frac{3^3(a^2)^3(b^6)^3}{4^3 \cdot a^3(b^3)^3} = \frac{27a^6b^{18}}{64a^3b^9} = \frac{27a^{6-3}b^{18-9}}{64}$
 $= \boxed{\frac{27a^3b^9}{64}}$

$$5. \text{ a. } 5^2 = 5 \cdot 5 = \boxed{25} \quad \text{b. } -5^2 = -(5)(5) = \boxed{-25} \quad \text{c. } (-5)^2 = (-5)(-5) = \boxed{25}$$

$$\text{d. } -5^{-2} = -\frac{1}{5^2} = \boxed{-\frac{1}{25}}$$

$$6. \frac{2^0}{3^0 + 4^0} \quad (\text{remember: } (\text{any number})^0 = 1 \text{ except } 0^0)$$

$$= \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

$$7. \frac{(2x^{-3}y^4)^{-2}}{(3x^{-6}y^{-2})^3} = \frac{2^{-2}(x^3)^{-2}(y^4)^{-2}}{3^3(x^{-6})^3(y^{-2})^3} = \frac{2^{-2}x^6y^{-8}}{27x^{-18}y^{-6}}$$

(Factors with negative exponents are moved to the other side of the fraction and the exponent is changed to +.)

$$= \frac{x^6 \cdot x^{18} \cdot y^6}{2^2 \cdot 27 \cdot y^8}$$

$$= \frac{x^{6+18} y^6}{4 \cdot 27 y^8} = \frac{x^{24}}{108 y^{8-6}} = \boxed{\frac{x^{24}}{108 y^2}}$$

$$\text{or } \frac{x^{24} y^{6-8}}{108} = \frac{x^{24} y^{-2}}{108} = \frac{x^{24}}{108 y^2}$$

$$8. 4^{-1} + 5^{-1} = \frac{1}{4} + \frac{1}{5} = \frac{1 \cdot 5}{4 \cdot 5} + \frac{1 \cdot 4}{5 \cdot 4} = \frac{5}{20} + \frac{4}{20} = \boxed{\frac{9}{20}}$$

$$9. (x^2 - y^2)^2 = (x^2 - y^2)(x^2 - y^2) = x^2(x^2 - y^2) - y^2(x^2 - y^2)$$

$$= x^4 - x^2y^2 - x^2y^2 + y^4$$

$$= \boxed{x^4 - 2x^2y^2 + y^4}$$

$$10. \frac{0^{10}}{10^0} = \frac{0}{1} = \boxed{0} \quad (0^{\text{any power}} = 0 \text{ except } 0^0 \text{ is undefined})$$

$$11. (4x^{-2}y)^{-2} (2xy^{-3})^3 = 4^{-2} (x^{-2})^{-2} y^{-2} \cdot 2^3 x^3 (y^{-3})^3$$

$$= 4^{-2} x^4 y^{-2} \cdot 8 x^3 y^{-9} = \frac{x^4 \cdot 8 \cdot x^3}{4^2 y^2 \cdot y^9}$$

$$= \frac{1}{2} \frac{8x^7}{y^{11}} = \boxed{\frac{x^7}{2y^{11}}}$$

$$12. \frac{(3a^{-2}b^3)^2}{(a^3b^{-2})^{-3}} \cdot (5a^{-1}b^{-4})^{-1} = \frac{3^2 (a^{-2})^2 (b^3)^2}{(a^3)^{-3} \cdot (b^{-2})^{-3}} \cdot \frac{5^{-1} (a^{-1})^{-1} (b^{-4})^{-1}}{1}$$

$$= \frac{9a^{-4} b^6}{a^{-9} b^6} \cdot \frac{5^{-1} a b^4}{1} = \frac{9a^9 b^6}{a^4 \cdot b^6} \cdot \frac{ab^4}{5}$$

$$= \frac{9a^{9-4} b^{6-6}}{1} \cdot \frac{ab^4}{5} = \frac{9a^5 b^0 \cdot a^1 b^4}{5}$$

$$= \boxed{\frac{9a^6 b^4}{5}}$$

Additional Resources

Click on the links below to download worksheets for more practice:

<https://cdn.kutasoftware.com/Worksheets/Alg1/Properties%20of%20Exponents.pdf>

<https://cdn.kutasoftware.com/Worksheets/Alg1/More%20Properties%20of%20Exponents.pdf>