DIY: Factoring Polynomials- A review

To review Factoring Polynomials, watch the following set of YouTube explaining the basic techniques for factoring polynomial expressions starting with "finding GCF", followed by 20 factoring practice problems for you to try covering all the basic techniques, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. Finding the greatest common factor (GCF)
2. Factoring Using the GCF, Ex 1
3. Factoring Using the GCF, Factoring out binomials
4. Factoring by Grouping – Ex. 1

   Factoring by grouping – Ex 2
5. Factoring Trinomials by Trial and Error
6. Factoring Trinomials: Factor by Grouping – Ex 1

   Factoring Trinomials: Factor by Grouping – Ex 2

   Factoring Trinomials: Factor by Grouping – Ex 3
7. Factoring the Difference of Two Squares – Ex 1

   Factoring the Difference of Two Squares – Ex 2

   Factoring the Difference of Two Squares – Ex 3
8. Factoring Perfect Square Trinomials – Ex 1

   Factoring Perfect Square Trinomials – Ex 2
9. Factoring Sums and Differences of Cubes

Practice problems: The following twenty problems use the techniques demonstrated in the above videos. The answers are given after the twenty problems. Then detailed solutions, if you need them, are given after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Finding GCF: What is the greatest common factor among these 3 numbers: 18, 24, 42 __________
2. Find 18 + 24 – 42 by first factoring out the GCF, then simplifying. __________
3. Factor: $8x - 12y + 16z =$ ____________
4. Factor: $9x^3 - 3x^2 + 12x =$ ___________________
5. Factor: $x^3y^4 - 7x^2y^5 + 9xy^6 =$ ________________________

6. Factor: $6x^2(x+y) - 9x^3(x+y)$ = ____________________

7. Factor by grouping: $6a^2 + 9ab - 8ab - 12b^2$

8. Factor by grouping (hint: re-arrange the terms first): $28a + 3b + 7ab + 12 =$ ____________________

9. Factor: $xy + 5x - y - 5 =$ ________________________

10. Factor: $4xy - 24y - 20x + 120 =$ ______________________

11. Factor the trinomial: $x^2 - 2x - 24 =$ __________________

12. Factor: $6x^2 + 26xy + 20y^2 =$ ___________________

13. Factor: $x^2 - 4x + 6$ = _____________________

14. Factor using the factor by grouping method: $24x^2 + 14xy - 24y^2 =$ ____________________

15. Factor: $24x^4y - 96x^6y =$ _________________________

16. Factor: $16x^2 - 80x + 100 =$ ____________________

17. Factor: $a^4 - 16b^4 =$ ___________________

18. Factor: $a^9 + 64b^3$ = ____________________

19. Factor: $(4x-3)^2 - (3x+2)^2 =$ ______________________

20. Factor: $10(2x+3)^2 - 13(2x+3) + 3$ = ____________________ (Hint: substitute u = 2x+3, factor in terms of u, then back substitute 2x+3 for u, and simplify).

Answers:

1. 6
2. 0
3. $4(2x - 3y + 4z)$
4. $3x(3x^2 - x + 4)$
5. $xy^4(x^2 - 7xy + 9y^2$
6. $3x^2(x+y)(2-3x)$
7. $(3a - 4b)(2a + 3b)$
8. $(7a+3)(b+4)$
9. $(x-1)(y+5)$
10. $4(x-6)(y-5)$
11. $(x-6)(x+4)$
12. $2(x+y)(3x+10y)$
13. Prime (can't be factored)
14. $2(4x-3y)(3x+4y)$
15. $24x^4y(1-2x)(1+2x)$ or $-24x^4y(2x-1)(2x+1)$
16. $4(2x-5)^2$
17. $(a-2b)(a+2b)(a^2+4b^2)$
18. $(a^3+4b)(a^6 - 4a^3b + 16b^2)$
19. $(7x-1)(x-5)$
20. $2(x+1)(20x+27)$

Detailed Solutions to Factoring Problems

1. $18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3$ or $2 \cdot 3^2$
$24 = 2 \cdot 12 = 2 \cdot 2 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 3$ or $2^3 3$
$42 = 6 \cdot 7 = 2 \cdot 3 \cdot 7$
GCF of $2 \cdot 3^2,\ 2^3 3,\ 2 \cdot 3 \cdot 7 = 2^1 \cdot 3^1 = \boxed{6}$

2. $18 + 24 - 42 = 6(3) + 6(4) - 6(7) = 6(3+4-7) = 6(0) = \boxed{0}$

3. $8x - 12y + 16z = 4(2x) - 4(3y) + 4(4z) = \boxed{4[2x - 3y + 4z]}$

4. $9x^3 - 3x^2 + 12x = 3x(3x^2 - x + 4)$

Can $3x^2 - x + 4$ be factored more? $3(4) = 12$. Can we find 2 numbers that multiply to 12 and add to -1? no.
So $\boxed{3x(3x^2 - x + 4)}$ is the final factored form.

5. $x^3y^4 - 7x^2y^5 + 9xy^6$. The numerical coefficients — 1, -7, and 9 — have no common factors other than 1. The smallest power of x is x^1 and the smallest power of y is y^4, so the GCF is xy^4.

$$\boxed{xy^4(x^2 - 7xy + 9y^2)}$$

Can $x^2 - 7xy + 9y^2$ be factored further? No.
$(x-3y)(x-3y) = x^2 - 6xy + 9y^2$
and $(x-y)(x-9y) = x^2 - 10xy + 9y^2$

6. $6x^2(x+y) - 9x^3(x+y) = (x+y)(6x^2 - 9x^3)$
$= (x+y)[3x^2(2-3x)] = \boxed{3x^2(x+y)(2-3x)}$

7. $\underbrace{6a^2 + 9ab}\ \underbrace{-8ab - 12b^2}$
$= 3a(2a+3b) - 4b(2a+3b)$

Factoring out the binomial $(2a+3b)$ gives $\boxed{(2a+3b)(3a-4b)}$

8. $28a + 3b + 7ab + 12 = \underbrace{28a + 7ab} + \underbrace{3b + 12}$ or $\underbrace{28a + 12} + \underbrace{3b + 7ab}$

$= 7a(4+b) + 3(b+4)$

$= \boxed{(4+b)(7a+3)}$

$= 4(7a+3) + b(3+7a)$

$= \boxed{(4+b)(7a+3)}$

9. $\underbrace{xy + 5x} - \underbrace{y - 5} = x(y+5) + (-1)(y+5) = x(y+5) - 1(y+5)$

$= \boxed{(x-1)(y+5)}$

10. $\underbrace{4xy - 24y} - \underbrace{20x + 120} = 4y(x-6) + 20(-x+6)$

$= 4y(x-6) - 20(x-6)$

$= (4y-20)(x-6)$

$= \boxed{4(y-5)(x-6)}$

Note: The "4" could have been factored out of all four terms at the first step.

11. $x^2 - 2x - 24$ Look for 2 numbers that
- multiply to -24
- add to -2

$\Big\}$ $4, -6 \longrightarrow \boxed{(x+4)(x-6)}$

12. $6x^2 + 26xy + 20y^2 = 2\left[3x^2 + 13xy + 10y^2\right]$

Because the coefficient of x^2 is not 1, the "trial and error" method becomes less efficient than the method using grouping.

1. multiply $(3)(10) = 30$
2. find two numbers that
   - multiply to 30
   - add to 13

   $\Big\}$ 3, 10
3. Rewrite $13xy$ as $3xy + 10xy$
4. Factor by grouping: $2\left[3x^2 + 3xy + 10xy + 10y^2\right]$

$= 2\left[3x(x+y) + 10y(x+y)\right] = \boxed{2(3x+10y)(x+y)}$

13. x^2-4x+6

Find 2 numbers that
- multiply to $+6$
- add to -4 (so both numbers must be negative)

possible number pairs. $(-1)(-6)=6$ but $(-1)+(-6)=-7$
$(-2)(-3)=6$ but $(-2)+(-3)=-5$

neither of the possible number pairs gives a correct middle coefficient so we conclude that x^2-4x+6 is <u>not factorable.</u> It is $\boxed{\text{prime}}$

14. $24x^2+14xy-24y^2 = 2(12x^2+7xy-12y^2)$

$(12)(-12)=-144$ Need 2 numbers that
- multiply to $-144 \rightarrow$ 1 positive 1 negative
- add to $+7 \rightarrow$ large number is $+$

possible number pairs:

$-1, 144$	adds to	143
$-2, 72$	"	70
$-3, 48$	"	45
$-4, 36$	"	32
$-6, 24$	"	18
$-8, 18$	"	10
$-9, 16$	"	7

$$= 2\left[\underbrace{12x^2-9xy}+\underbrace{16xy-12y^2}\right]$$

$$= 2\left[3x(4x-3y)+4y(4x-3y)\right]$$

$$= \boxed{2(3x+4y)(4x-3y)}$$

15. $24x^4y - 96x^6y = 4x^4y\left[6 - 24x^2\right] = 4x^4y \cdot 6\left(1 - 4x^2\right)$

$= 24x^4y\left(1^2 - (2x)^2\right)$ (difference of squares)

$= \boxed{24x^4y\,(1-2x)(1+2x)}$

16. $16x^2 - 80x + 100 = 4(4x^2 - 20x + 25)$

Since both the leading coefficient (of x^2) and the constant are perfect squares, check to see if this is a perfect square trinomial.

$(a+b)^2 = a^2 + 2ab + b^2$ or $(a-b)^2 = a^2 - 2ab + b^2$

For $4x^2 - 20x + 25$

$a^2 = 4$, $a = 2$ $\qquad$ $b^2 = 25$, $b = 5$

$-2ab = -2(2)(5) = -20$ ← Since this is the coefficient of middle term, this <u>is</u> a perfect square trinomial.

$4\left(4x^2 - 20x - 25\right)$

$= \boxed{4(2x-5)^2}$

Note: this trinomial could also be factored using trial and error or grouping.

17. $a^4 - 16b^4 = (a^2)^2 - (4b^2)^2 = (a^2 - 4b^2)(a^2 + 4b^2)$

The first factor can be factored further:

$= \left(a^2 - (2b)^2\right)\left(a^2 + 4b^2\right)$

$= \boxed{(a-2b)(a+2b)(a^2+4b^2)}$

Note: $a^2 + 4b^2$ is the <u>sum</u> of squares and is not factorable.

18. $a^9 + 64b^3 = (a^3)^3 + (4b)^3$

Use the sum of cubes factoring pattern: $x^3+y^3 = (x+y)(x^2-xy+y^2)$

$$(a^3)^3 + (4b)^3 = (a^3+4b)\left((a^3)^2 - (a^3)(4b) + (4b)^2\right)$$
$$= \boxed{(a^3+4b)(a^6-4a^3b+16b^2)}$$

19 $(4x-3)^2 - (3x+2)^2$ could be factored by first multiplying out, simplifying, then factoring.

Can also factor using $a^2-b^2 = (a-b)(a+b)$

$$(4x-3)^2 - (3x+2)^2 = \left((4x-3)-(3x+2)\right)\left((4x-3)+(3x+2)\right)$$
$$= (4x-3-3x-2)(4x-3+3x+2)$$
$$= \boxed{(x-5)(7x-1)}$$

20. $10(2x+3)^2 - 13(2x+3) + 3$

If we begin by multiplying out: $10(4x^2+12x+9) - 13(2x+3) + 3$

$$= 40x^2 + 120x + 90 - 26x - 39 + 3$$
$$= 40x^2 + 94x + 54$$
$$= 2(20x^2 + 47x + 27)$$

This will be difficult to factor since the coefficients are large

Try making a substitution in the original expression.
If $u = 2x+3$ then
$$10u^2 - 13u + 3$$

Using grouping method: $10(3) = 30$
$(-10)(-3) = 30$ and $(-10)+(-3) = -13$

$$10u^2 - 10u - 3u + 3$$
$$= 10u(u-1) - 3(u-1) = (10u-3)(u-1)$$

back substituting: $= \left(10(2x+3)-3\right)(2x+3-1) = (20x+30-3)(2x+2)$
$$= (20x+27)(2x+2) = \boxed{2(x+1)(20x+27)}$$

Additional Resources

Click on the links below to download worksheets for more practice:

1. Factoring quadratic polynomials (easy, hard)
2. Factoring special case polynomials
3. Factoring by grouping

Alternatively;

1. Go To http://www.kutasoftware.com/free.html
2. Under "**Polynomials**" click on:
   - Factoring quadratic polynomials (easy, hard)
   - Factoring special case polynomials
   - Factoring by grouping
3. You can print out the worksheets and work on them.
4. For help please contact the ***Math Assistance Area***.