DIY: *Significant Digits*

To review the Significant Digits, watch the following YouTube video. The video goes over what are significant digits and performing arithmetic operations on them. They are followed by several practice problems for you to try, covering all the basic concepts covered in the video, with answers and detailed solutions. Some additional resources are included for more practice at the end.

- Significant digit basics and arithmetic with significant digits
https://www.youtube.com/watch?v=6oj4y0d44nQ
- More on adding/subtracting with significant digits
https://www.youtube.com/watch?v=xHgPtFUbAeU
- More on significance of zeros   https://www.youtube.com/watch?v=7b60RZqut0U
- Why significant digits are important https://www.youtube.com/watch?v=VAuslY-Uuf4
- Exact numbers vs. measured numbers:
https://www.youtube.com/watch?v=WT_6g3NCClE

Practice problems: The following problems use the techniques demonstrated in the above video. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact *Math Assistance Area*.

1. Write the number of significant digits for the following
   a) 26.36
   b) 5601.203
   c) 69,000
   d) 9000.10
   e) 45.36×10^{-5}
   f) 1.08×10^{10}
   g) 9.8560×10^{-8}
   h) 2.0×10^{23}
2. Perform the indicated operation and write your answer in appropriate significant digits.
   a) 0.2364 +1.020
   b) 1.6502 – 5.36
   c) 2501.30 ÷ 605.00
   d) 45.0 +30.0
   e) 36.20 ×1.23
   f) 800 – 52
   g) 0.58 ×2
   h) 3.69 ÷ 10
3. a) What is the area of a circle with radius 30.205 m? Write the answer in appropriate significant digits.(*Hint*: $A = \pi r^2$)   b) If this circle is to be constructed using material that costs $49.75 per square meter, what is the cost of the circle? (Assume the material cost is rounded to the nearest cent).
4. A projectile shot in air from the ground at initial velocity of 109.30 ft/s. Find the height of the projectile in 5 seconds, if it follows the following path
   $h (t)$ = -16 t^2 +$v_0 t$
   where v_0 is the initial velocity of the projectile

Answers:

1.
 a) 4   b) 7   c) 2*   d) 6   e) 4   f) 3   g) 5   h) 2
2.
 a) 1.256   b) -3.71   c) 4.1344   d) 75.0   e) 44.5   f) 700   g) 1.2 or 1 or 1.16*   h) 0.4 or 0.369*

3. a) 2866.2 m^2 4. 150 or 146.50 feet*   * see detailed solutions for explanation
b) $142,600

Detailed Solutions

1. a. $\underline{26.36}$ has ④ significant digits.

   b. $\underline{5601.203}$ has ⑦ significant digits (s.d.) All non-zero digits are significant, and zeros that occur between non-zero digits are significant also.

   c. $\underline{69},000$ According to the guide lines from the videos, this would have ② significant digits. The zeros to the right of the last non-zero digit (9) would not be considered significant in the absence of a decimal point. However, it is ambiguous in this form since the original measurement could have been 69,002, so 69,000 would be correct to the tens digit.

   For numbers ending in zeros without a decimal point, use scientific notation to remove ambiguity
   $69,000 = 6.9 \times 10^4$ (2 s.d.)
   $= 6.900 \times 10^4$ (4 s.d.)

   d. $\underline{9000.10}$ has ⑥ significant digits — 2 non-zero digits, 3 "sandwiched" zeros, and a trailing zero to the right of the decimal point.

   e. $\underline{45.36} \times 10^{-5}$ has ④ significant digits. If this number were written without the power of 10 it would be $.0000\underline{4536}$. In this form, the 4 zeros to the right of the decimal point are <u>not</u> significant since they are not to the right of all non-zero digits. These 4 zeros are needed for place values only.

   Note: Leading zeros (to the left of all non-zero digits) are never significant, so .00004536 could be written as 0.00004536, still with only 4 significant digits. The leading zero to the left of the decimal point is often included to call attention to the decimal point.

   f. $\underline{1.08} \times 10^{10}$ has ③ significant digits. Note that the "× 10" adds no significant digits. It's purpose is to establish place value. Written without the power of 10, this number would be 10800000000 with no decimal point shown. Ending zeros (with no non-zero digits or decimal point to the right) are <u>not</u> significant.

1.g. 9.8560×10^{-8} has 5 significant digits. The terminating zero to the right of the decimal is significant.

h. 2.0×10^{23} has ② significant digits

2.a. $0.2364 + 1.020 = $

$$0.2364$$
$$+ \underline{1.020}$$
$$1.256|4 \rightarrow \boxed{1.256}$$

The sum/difference of numbers can have no more decimal places than the least number of decimal places in the numbers being added/subtracted.

b. $1.6502 - 5.36 = $

$$1.6502$$
$$-\underline{5.36} \qquad \text{or} \qquad -\left(\begin{array}{c} 5.36 \\ -\underline{1.6502} \end{array}\right)$$

$$= -\left(\begin{array}{c} 5.36\,^4\,^{5}6\,^9\,0 \\ -\underline{1.6502} \end{array}\right) \leftarrow \begin{array}{l}\text{these zeros are added to make the subtraction easier.}\end{array}$$

$$-\;3.7098$$

$$= -3.70|98 \approx \boxed{-3.71}$$

5 or greater, so round up.

Answer can only have 2 decimal places since that is the least number of decimal places in the input numbers.

c. $\underline{2501.30} \div \underline{605.00} = 4.13438017 \approx \boxed{4.1344}$

6 s.d.   5 s.d   can only have 5 s.d.

s.d. = significant digits

d. $\underline{45.0} + \underline{30.0} = \boxed{75.0}$

Significant.

Since both numbers being added are precise to the nearest tenth, the answer will be given to the tenths.

But note: If the problem were $45.0 + 30$, the answer would be 80. 30 is only precise to the tens, so 75.0 would be rounded to the nearest ten: $75.0 \rightarrow 80$.

2.e. 36.20×1.23
    $\quad$ 4 s.d. $\quad$ 3 s.d. → answer should have
    $\quad\quad\quad\quad\quad\quad\quad$ 3 signif. digits

$= 44.5\overset{|}{2}6 \approx \boxed{44.5}$

$\quad\quad\quad\quad$ first non-significant digit
$\quad\quad\quad\quad$ is less than 5, so <u>don't</u>
$\quad\quad\quad\quad$ round up.

f. $800 - 52 = 748$
$\quad\quad\quad\quad \approx \boxed{700}$

$\quad\quad$ 800 is precise only to the hundreds.
$\quad\quad$ 52 is precise to the ones (units).
$\quad\quad$ Therefore, our answer can only be precise
$\quad\quad$ to the hundreds.

g. 0.58×2 ← note: In this case, without any further context
$\quad\quad$ 2 s.d. $\quad\quad$ to the problem, it is ambiguous as to whether
$\quad\quad\quad\quad\quad\quad\quad$ the "2" is a measured quantity (with
$= 0.58$ $\quad\quad\quad\quad$ only 1 significant digit) or an exact number
$\quad\underline{2}$ $\quad$ → $\quad$ (meaning we are doubling the 0.58. For example,
$\quad 1.16$ $\quad\quad\quad\quad$ an item costs $\$.58$ and one buys 2 items.)

$\quad$ • In this case, an argument could be made for an
answer with 1 significant digit (assuming 2 is a
measured quantity), so the answer would be 1 or 1.
$\quad\quad$ • It may also be reasonable to interpret the "2"
as an exact number (infinite number of significant digits), in
which case we should interpret "0.58×2" as "0.58 + 0.58"
$\quad\quad\quad\quad\quad\quad\quad\quad\quad = \boxed{1.16}$ (2 decimal places).
$\quad\quad\quad\quad$ (buying 2 items at $\$.58$ each would cost $\$1.16$).
$\quad$ • However, the best answer may be $\boxed{1.2}$
$\quad\quad$ If 2 is exact, then 0.58 has 2 significant digits and
the product would have 2 s.d → $1.16 \approx 1.2$
$\quad\quad\quad$ To show the sense of this: 0.58 could be 0.575 to 0.584
$\quad\quad\quad$ in which case 2(0.58) could be any number 1.15 to 1.168,
$\quad\quad\quad$ so expressing the answer as 1.2 would be the best answer.
Note: In the "real world", this problem would have more information
$\quad\quad$ about the context, so it would be clearer which approach is best

2.h. $3.69 \div 10 = .369$ Again, if 10 is interpretted as an exact number, then the answer should have 3 s.d.
Ans. = .369 or $\boxed{0.369}$

If 10 is interpretted as a measured quantity, then it would have only 1 s.d. and the answer should also only have 1 s.d.   Ans. = .4 or $\boxed{0.4}$.

3. a. $A = \pi r^2 = \pi (30.205)^2 = 2{,}846.2070033\ldots$

↑
using calculator
"π" key is best.
π is an exact, but irrational number, so if we used 3.14 for π, our answer would only have 3 significant digits.

As long as we used at least a 5-digit approximation for π, our answer should have 5 s.d.
$A \approx 2866.2 \text{ m}^2$

b.   Cost = area × cost/m² $= \pi (30.205)^2 (49.75)$
$= {}^{\$}142{,}593.7984142\ldots$
$\approx {}^{\$}142{,}600$

notes: 1. since the cost was specified to have been rounded, it has 4 s.d., radius has 5 s.d., and if the approximation used for π had at least 4 s.d., then the answer should have 4 s.d.   Cost ≈ $142,600

2. It would not the appropriate to calculate cost using the ~~rounded~~ area (Cost = ~~2866.2 × 49.75~~)
Although in this case the same 4 s.d. answer would result, in many cases, rounding intermediate results can cause significant error in a final calculation!

4. $h(t) = -16t^2 + v_0 \cdot t$ where $t = 5$ sec. and $v_0 = 109.30$ ft/sec.

In this problem, we need to identify which numbers are measured quantities and which can be assumed to be exact.

1. Assume $t = 5$ is exact. The question is to find the height 5 sec. after launch.

2. v_0 is measured, with 5 s.d.

3. "16" is actually $\frac{1}{2}$ acceleration on earth's surface due to gravity and is measured correct to 2 s.d. in this problem.

To find the correct answer, we use multiplication rules to find the value of each term, to the correct number of s.d.

Then we use addition/subtraction rules to put the two terms together.

$$h(5) = \underbrace{-16 \, (5)^2}_{\text{to 2 s.d.}} + \underbrace{109.30(5)}_{\text{to 5 s.d.}}$$

$$= -400 + 546.50 \text{ ft.}$$

$$= -4.0 \times 10^2 + 5.4450 \times 10^2 \quad \text{using scientific notation to show that the first zero in 400 is actually significant but the second zero is not}$$

$$= 1.4650 \times 10^2 \quad (\text{need 2 s.d.})$$

$$\approx 1.5 \times 10^2$$

$$\text{or} \quad \boxed{150 \text{ feet}}$$

Note: If we did not know that "16" was not exact, then
$$h(5) = \underset{\text{exact}}{-400} + \underset{\text{5 s.d.}}{546.50} = \boxed{146.50 \text{ feet}}$$

Additional Resources

1. Go to
   https://www.saddleback.edu/faculty/jzoval/worksheets_tutorials/ch1worksheets/worksheet_Sig_Fig_9_11_08.pdf
   You can print out the worksheet and work on them. The solutions are provided at the end of the worksheet

2. For help please contact the *Math Assistance Area.*