Physics 1180

Section 1.3: Forces

Newton's Law of Universal Gravitation,
Planetary Motion & Coulomb's Law

As the story goes…

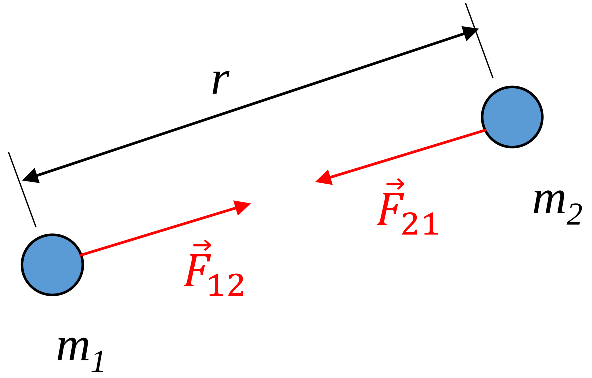
Before Isaac Newton, it was believed that the rules governing the motion of objects on Earth were different from those that governed the motions of the Moon and the planets. (After all, "terrestrial" objects fell to the ground and "celestial" objects stayed in the sky.)

One day, a young man was sitting under a tree while the moon was visible in the sky. Upon witnessing an apple fall from the tree, he pondered whether the force drawing the apple to the ground and the force that held the moon in its orbit were the same…

Newton's Law of Universal Gravitation

Mutual attraction between objects that have mass.

$$|\vec{F}_{12}| = |\vec{F}_{21}| = F_{grav} = \frac{Gm_1m_2}{r^2},$$



where $G = 6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$ is the *universal gravitational constant*.

The gravitational force obeys an *inverse-square law*. (If the distance between the two masses is doubled, the force of attraction will decrease by a factor of four. Triple the distance, and the force will become one-ninth.)

The Gravitational Field

Recall that we described *weight* as the "force of gravity" acting on an object: That is, $F_{grav} = mg$. Combine this with Newton's law of universal gravitation:

$$F_{grav} = mg = \frac{Gm_1m_2}{r^2}$$

With m_1 being the mass of the object, m_2 being the mass of the Earth (M_E), and r being the separation between the object and the center of the Earth (R_E):

$$mg = \frac{GmM_E}{R_E^2}.$$

Then $g = \frac{GM_E}{R_E^2} = \frac{(6.67\times10^{-11}\frac{\text{N.m}^2}{\text{kg}^2})(5.98\times10^{24}\text{ kg})}{(6.637\times10^{6}\text{ m})^2} = 9.83\text{ m/s}^2$.

Planetary Motion

When near the surface of the Earth, the acceleration due to gravity can be regarded as constant (g = 9.8 m/s^2 ≈ 10 m/s^2).

As seen on the previous slide, when far away from the Earth's surface (or any celestial body for that matter), the acceleration due to gravity, g, can no longer be treated as constant. It too obeys an inverse-square law.

As a result, for altitude that is 5% percent of the Earth's radius (about 200 miles of the surface), g = 8.9 m/s^2 ≈ 9 m/s^2.

The formula for GPE is not longer just mgh but is modified to be:

$$\text{Gravitational Potential Energy} = -\frac{GmM_E}{r}, \text{ where } r = R_E + h.$$

(The minus implies that the "zero energy reference" is taken at $r = \infty$.)

Consider Circular Orbits

Recall that for an object of mass m moving in a circle of radius r at at a (constant) speed v undergoes a centripetal acceleration and experiences a centripetal force given by, $F_{centripetal} = mv^2/r$. The centripetal force in this case is caused by gravity. Combine this with Newton's law of universal gravitation:

$$F_{centripetal} = F_{grav}$$

With m_1 being the mass of the object, m_2 being the mass of the Earth (M_E), and r being the separation between the object and the center of the Earth (R_E):

$$\frac{mv^2}{r} = \frac{GmM_E}{r^2}.$$

We can now solve for the orbital speed, $v = \sqrt{\frac{GM_E}{r}}$.

Circular Orbits (cont'd)

When near the surface of the Earth, the acceleration due to gravity can be regarded as constant (g = 9.8 m/s^2 ≈ 10 m/s^2).

As seen on the previous slide, when far away from the Earth's surface (or any celestial body for that matter), the acceleration due to gravity, g, can no longer be treated as constant. It too obeys an inverse-square law.

As a result, for altitude that is 5% percent of the Earth's radius (about 200 miles of the surface), g = 8.9 m/s^2 ≈ 9 m/s^2.

The formula for GPE is not longer just mgh but is modified to be:

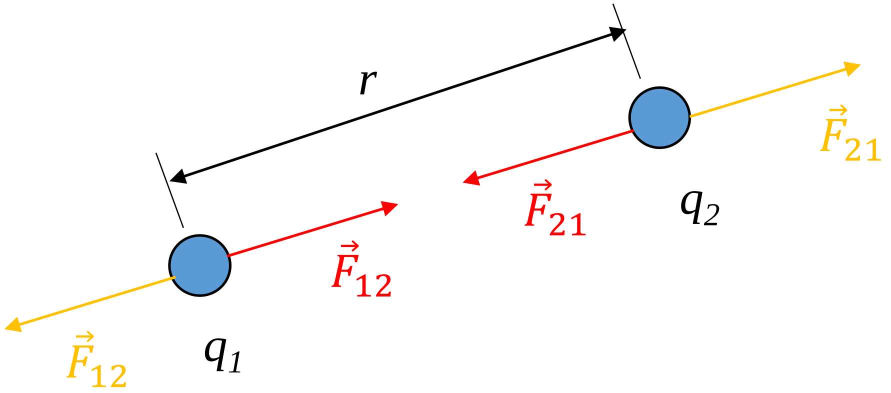
$$\text{Gravitational Potential Energy} = -\frac{GmM_E}{r}, \text{ where } r = R_E + h.$$

(The minus implies that the "zero energy reference" is taken at $r = \infty$.)

Coulomb's law

Mutual attraction or repulsion between objects that have electric charge.

$$|\vec{F}_{12}| = |\vec{F}_{21}| = F_{elec} = \frac{k|q_1||q_2|}{r^2},$$



where $k = 9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ is the *electrostatic constant*.

The electrical force has the same mathematical form as Newton's law of universal gravitation. It too obeys an *inverse-square law*. However, the electric force is MUCH stronger and can be repulsive as well as attractive.

"Like" charges repel and "opposite" charges attract.