Name: __________________________ Partner(s): __________________________

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A MECHANICAL ANALOGY TO THE MILLIKAN OIL DROP EXPERIMENT

In this investigation, you will perform a simulation of the famous "Millikan oil-drop experiment." In his experiment performed in 1909, American physicist Robert Millikan was able to determine the *elementary charge* of an electron. He did this by observing small electrically charged oil droplets as they rose and fell at terminal velocity under the influence competing electrical and gravitational forces. However, rather than trying to painstakingly observe the motion of tiny oil droplets to measure the elementary charge as Millikan did, you will perform the same basic methodology to determine the "elementary mass" of a particular hex nut under the influence of competing gravitational and buoyant forces. As a result, you will essentially observe the same physics in a hands-on fashion, but with much less eyestrain and experimental difficulty.

Background

Until the late 1890's, atoms were believed* to be the indivisible fundamental building blocks of matter. However, in 1897 English physicist Joseph John (J. J.) Thompson demonstrated that atoms did in fact consist of smaller electrically charged particles. He did so by heating gas atoms in a strong electric field. This resulted in ionization of the gas atoms and produced what was known at the time as *cathode rays*. By observing the deflection of these cathode rays using controlled electric and magnetic fields, Thompson's experiments led to the discovery that these cathode rays were streams of negatively charged particles that are now called *electrons*.

In the original experiment, Millikan's arrangement consisted of a cylindrical enclosure in which a mist of oil was sprayed and allowed to fall through a small opening under the influence of gravity at terminal velocity into a region between two plates that served as a capacitor to produce a vertical electric field. (See Fig. 1) An x-ray source was used to ionize molecules in the air of the chamber. The free electrons that resulted attached themselves to oil droplets falling through the chamber. By varying the voltage between the capacitor plates, the electric field could be made to cause a particular oil drop to rise or fall or remain stationary. The mass of any given oil drop was measured from the terminal velocities during the rise and fall of the oil drop. With the mass of the oil drop and voltage necessary to hold the oil drop stationary, the charge on the oil drop was measured.

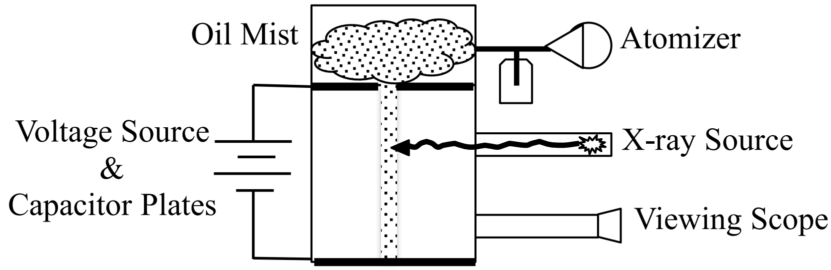


Fig. 1: Schematic layout of Millikan's apparatus.

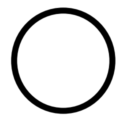
*Even in the early 20^{th} century, the existence of atoms was still a controversial issue.

Through repeated measurements of many oil drops, Millikan found that the electric charge on any given oil drop was always an integer multiple of a lowest value. This lowest value was the magnitude of electric charge for a single electron: the *elementary charge*, e, where

$$e = 1.602 \times 10^{-19}\ \text{C}.$$

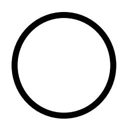
(Recall that electrons are negatively charged. That is, the charge of a single electron is actually $-e$.)

Questions: Consider an electrically charged oil drop of mass M that is moving between the capacitor plates of the chamber at its terminal velocity. What can be said about the *net force* that acts on the oil drop? In the space to the right, sketch a free-body diagram on the oil drop in this case. Be sure to label all the forces on your diagram.



In your diagram above, it is likely that you assumed no "frictional effects" on the oil drop due to the air in the chamber. However, in order to accurately determine the elementary charge, Millikan had to deal with a speed-dependent viscous drag of the oil drops in the air.

Questions: Redraw your free-body diagram for the oil drop to include viscous drag. Again, be sure to label all the forces on your diagram.



Question: Based on the direction that you indicated for the viscous drag on your free-body-diagram, is the oil drop in your second diagram a "floater" or a "sinker?" Explain.

Checkpoint: Consult with your instructor before proceeding. **Instructor's OK:** ________

In an analogous fashion to Millikan's experiment, you will observe small canisters that contain unknown numbers of hex nuts as they move at terminal velocity through a column of water in order to determine the mass of an individual hex nut. Refer to Table 1 to see the relationship between Millikan's experiment and the analogous mechanical simulation. A schematic layout of your experimental set-up is shown in Fig. 2.

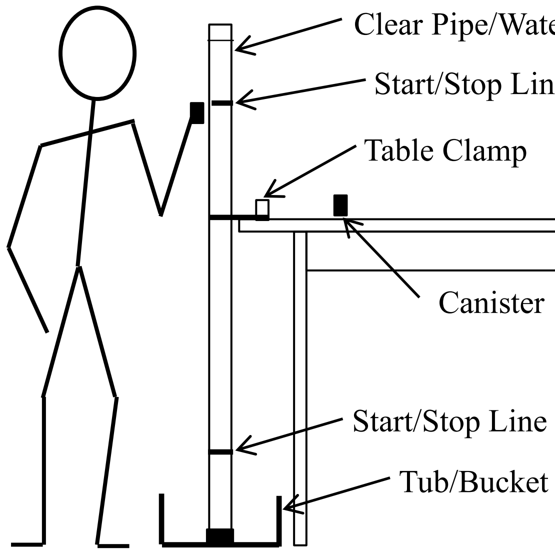


Fig. 2: Schematic layout of the apparatus used in the mechanical hex nut analogy to the Millikan oil drop experiment.

In this mechanical analogy, it should be noted that unlike Millikan's experiment in which the gravitational force varied as different oil drops had different masses, the buoyant force (which is analogous to Millikan's gravitational force) would be constant since all of the canisters are the same size. Refer to Table 1 below to see the relationship between Millikan's experiment and the analogous mechanical simulation.

Table 1

Millikan's Oil Drop Experiment	**Mechanical Analogy**
Electron Charge	Hex Nut Mass
Oil Drop	Plastic Canister
Gravitational Force	Buoyant Force
Electric Force	Gravitational Force
Viscous Air Drag	Viscous Water Drag

Question: Why is the electric force in Millikan's experiment analogous to the gravitational force (rather the buoyant force) in the mechanical experiment?

Checkpoint: Consult with your instructor before proceeding. **Instructor's OK:** _______

Starting with Newton's 2nd law of motion, we can write the vector sum of all the forces acting on the canister as the mass M of the canister times the acceleration a. That is,

$$\sum \vec{F} = \vec{F}_{bouy} + \vec{F}_{grav} + \vec{F}_{drag} = M\vec{a} \quad . \qquad (1)$$

At terminal speed v_t, the acceleration is zero. Rewriting the force vector equation as a scalar equation, we have

$$F_{bouy} - F_{grav} \pm F_{drag} = 0 \quad , \qquad (2)$$

where it is assumed that "up" is taken to be the $+y$-direction. The choice of the $\pm$ is determined by the whether the canister is a "sinker" a or a "floater."

Question: In the above equation involving F_{drag}, which sign (+ or –) is associated with the sinkers and which sign is associated with the floaters? Explain your reasoning.

According to Archimedes' principle, the upward buoyant force had a magnitude equal to the weight of the fluid displaced by the submerged object. If you have already covered buoyancy in your physics studies, you may recall that the magnitude of the upward buoyant force on an object immersed in a fluid is given by

$$F_{buoy} = \rho_{fluid} V_{sub} g \qquad (3)$$

where ρ_{fluid} is the density of the fluid, V_{sub} is the volume of the submerged portion of the object, and g is the familiar acceleration due to gravity (9.8 m/s^2). In this case, ρ_{fluid} is just the density of water (1000 kg/m^3) and the submerged portion of the canister is the full volume of the canister V_{can}, since the entire canister is submerged during the measurements.

For the purpose of this investigation, we will assume[1] that the magnitude of the viscous drag is proportional to v^2. That is, $F_{\text{drag}} = Cv^2$, where C is the "effective drag coefficient" that accounts for the density of the fluid as well as the shape and cross-sectional area of the canister. In order to determine the "elementary mass" of the hex nuts used in this experiment, you will first have to determine a numerical value for C.

With the total weight of the canister (force of gravity) equal to Mg, the force equation becomes

$$\rho_{water} V_{can} g - Mg \pm C v_t^2 = 0 \quad . \qquad (4)$$

Questions: Rearrange Eqn. (4) to formulate an expression for the effective drag coefficient *C*, in terms of all the other measureable variables. What are the units of *C*?

Checkpoint: Consult with your instructor before proceeding. **Instructor's OK:** _______

Part I
Determination of Effective Drag Coefficient

Your group will need the following materials/equipment for this part:

- 1 clear 5-cm diameter pipe at least 1.5 m long (water-filled and fitted with stoppers)
- 1 tall ring stand and clamps (to secure the pipe vertically)
- 1 bucket/tub (to minimize spillage)
- 1 meterstick/tape
- 1 stopwatch
- 1 strong neodymium magnet
- several identical water-tight canisters with varying masses of sand (or metal shot)
- 1 beaker (or small tub) of water
- 1 graduated cylinder

Suggestions for best results:

- Any air bubbles attached to the canister can have a significant effect on the buoyancy of the canister and therefore, the terminal speed. For consistency, be sure to shake off air bubbles attached to the canister prior to release.
- In order to achieve the most accurate results, you will need to note the location of same part each canister at "eye level" at it travels through the pipe. Be sure to move up and down with the canister so that your eyes are even with the starting and stopping lines.
- Due to the fact that the canisters may wobble or fishtail as they sink or rise, it is important to measure each canister several times in order to determine an average for its terminal speed.
- The shape of the leading end of the canister will affect the value of the effective drag coefficient. The same end of the canister must "lead" whether the canister is a sinker or a floater. For this experiment, the sinkers are turned up side down and released from the top while the floaters are carefully dragged to the bottom of cylinder right side up with a strong neodymium magnet and then released.

Procedure

1. Using the canisters specially marked for **Part I**, separate the "floaters" from the "sinkers" by placing them one at a time in a beaker of water and observing their behavior. Record the mass of each sinker in Table 1 and each floater in Table 2 below.
2. For a sinker, turn the canister up side down and hold in place at the top of the pipe. Be sure to shake off any air bubbles as these can greatly affect the terminal speed.
3. Release the canister. When the bottom of the canister (which is now on top!) crosses the top line, start the stopwatch. When that same part of the canister crosses the lower line, stop the stopwatch. Record the time of the fall between the two marked lines on the pipe in the first row of Table 2. Be sure to view each event at eye level.
4. Use the magnet to carefully pull the canister back to the top.
5. Repeat steps 2-4 for the same sinker at least three more times and complete the first row of Table 1. If an outlier occurs, identify it as such and repeat the measurement.
6. Repeat Steps 2-5 for at least 3 more sinkers and record your results in Table 2.

Table 2: Fall times for the "Sinkers"

"Sinker" Mass (kg)		Trial 1 t_1 (s)	Trial 2 t_2 (s)	Trial 3 t_3 (s)	Trial 4 t_4 (s)	Average t_{ave} (s)
M_1						
M_2						
M_3						
M_4						

7. For a floater, be sure to keep the floaters right side up and remember to shake off any air bubbles attached to the canisters. Carefully, drag the floater to the bottom of the pipe using the neodymium magnet.
8. Release the canister. When the bottom of the canister crosses the lower line, start the stopwatch. When that same part of the canister crosses the upper line, stop the stopwatch.
9. Record the rise time between the two marked lines on the pipe in the first row of Table 3.
10. Use the magnet to carefully pull the canister back to the bottom.
11. Repeat steps 7-9 for the same floater at least three more times and complete the first row of Table 3. If an outlier occurs, identify it as such and repeat the measurement.
12. Repeat Steps 7-11 for at least 3 more floaters and record results in Table 3.

Table 3: Rise times for the "Floaters"

"Floater" Mass (kg)		Trial 1 t_1 (s)	Trial 2 t_2 (s)	Trial 3 t_3 (s)	Trial 4 t_4 (s)	Average t_{ave} (s)
M_1						
M_2						
M_3						
M_4						

13. Refer back to Equation (4) and note the square of the terminal speed, v_t^2, is linear with the mass of the canister, M. That is, $v_t^2 = (\text{slope})*M + (y\text{-intercept})$.
14. Using the distance of 1.50 m between the starting and stopping lines, calculate the average terminal speed, v_t, and its square, v_t^2, for each sinker and floater. Complete Table 4.

Table 4: Terminal Speed

Sinker	Term. Speed v_t (m/s)	Term. Speed v_t^2 (m^2/s^2)	Floater	Term. Speed v_t (m/s)	Term. Speed v_t^2 (m^2/s^2)
1			1		
2			2		
3			3		
4			4		

Question: Starting with Eqn. (4), solve for v_t^2 in terms of the other variables. Be careful that you use the correct sign (+ or –) for the Cv_t^2 term in your calculation for the sinkers and the floaters. In terms of variables, what are the magnitudes of the slopes for the sinker data and the floater data?

Questions: In principle, the average drag coefficients for the sinkers and the floaters should be about the same. Why? If they were significantly different (>10%) to what do attribute the difference?

15. Using any available data analysis software, create separate plots of v_t^2 vs. M for the sinkers and for the floaters. (The graphs should be fairly straight lines.)
16. Using the fit routine of the analysis software, separately fit your sinker data and floater data to straight lines. From the linear fit, record the slopes of both lines.
17. Separately calculate the drag coefficients for the sinkers and for the floaters based on the slopes of the lines that you fitted with the analysis software. Don't forget the units!

C_{sink} = ______________ C_{float} = ______________

Checkpoint: Discuss the results of your v_t^2 vs. M graph and your drag coefficients with your in instructor before proceeding.

Instructor's OK: __________

18. Upon instruction approval, print the graphs with the slope values and include them with your lab packet. Be sure to fully annotate each graph.

Part II
Determination of Hex Nut Mass

In **Part I**, you exploited the linear relationship between the square of the terminal speed (v_t^2) and the total canister mass (M) to determine the effective drag coefficient (C) to account for the viscous drag on the canister due to the water. The total mass of the canister is the mass of the empty canister plus the mass of an integer number of hex nuts. That is,

$$M = M_{can} + Nm_{nut}\ , \tag{5}$$

where M_{can} is the mass of the empty canister, m_{nut} is the mass of a single hex nut, and N is an integer. Since the total canister mass is linear with the hex nut mass (m_{nut}), it must be true that v_t^2 also varies linearly with m_{nut}.

Just as you did in **Part I**, you will measure the terminal speed for the canisters that contain various numbers of hex nuts. The only difference now is that we already know the drag coefficients for the sinker and floaters. Since N can only take on integer values, repeated measurements of various canisters should result in v_t^2 data that tends to "clump" in groups in accordance to the number of hex nuts inside the canisters.

After you measure the terminal velocities for the canisters, you will create a *histogram* (i.e., a frequency distribution of occurrences) for the square of the terminal speed, v_t^2 for both the sinkers and floaters. Assuming careful measurements, the v_t^2 values data should indeed "luster in groups in accordance to the number of hex nuts inside the canisters. By assigning integer values to the groups based on the values of v_t^2, you will eventually be able to plot the square of the terminal speed, v_t^2 vs. N for both the sinkers and the floaters that should be a straight line. This may require some trial and error. The slope of the sinker and floater lines will then used to determine "elementary mass" of the hex nut.

Questions: Substitute the expression for the total mass of the canister from Eqn. (5) into for v_t^2 that you derived in **Part I** (below Table 4) to show that v_t^2 is linear with m_{nut}. That is, v_t^2 = (slope)*m_{nut} + (y-intercept). In terms of variables, what is the expression for the slope of this line?

Checkpoint: Consult with your instructor before proceeding. **Instructor's OK:** _______

Procedure

1. Using the canisters specially marked for **Part II**, repeat the same procedure as you did in **Part I** except for the following differences:

- You will have access to only one canister at a time. After you acquire all the data for that canister, you will return it in exchange for another.
- You will measure sinkers and floaters until you until you have at least five measurements of v_t for the sinkers and five for the floaters.

2. Complete Tables 5 and 6 for the average fall/rise times for the canisters just as you did in **Part I**.

Table 5: Fall times for the "Sinkers"

"Sinker"	Trial 1 t_1 (s)	Trial 2 t_2 (s)	Trial 3 t_3 (s)	Trial 4 t_4 (s)	Average t_{ave} (s)
1					
2					
3					
4					
5					

Sinker Drag Coefficient, C_{sink} = _______ (   ) (from **Part I**)

Table 5: Rise times for the "Floaters"

"Floater"	Trial 1 t_1 (s)	Trial 2 t_2 (s)	Trial 3 t_3 (s)	Trial 4 t_4 (s)	Average t_{ave} (s)
1					
2					
3					
4					
5					

Floater Drag Coefficient, C_{float} = ________ (   ) (from **Part I**)

3. Complete Table 7 for the terminal speeds and the square of the terminal speeds for your sinkers and floaters.

Table 7: Terminal Speed for Square of Terminal Speed

Sinker	v_t (m/s)	v_t^2 (m^2/s^2)	Floater	v_t (m/s)	v_t^2 (m^2/s^2)
1			1		
2			2		
3			3		
4			4		
5			5		

Question: Based on your results, does it appear that each of your canisters had a different number of hex nuts or did any appear to have the same number as another? What is the evidence for your answer?

4. Note the occurrences of v_t^2 for the sinkers and again for the floaters. For each of the sinkers and floaters, assign $N = 1$ to the lowest "clump" of v_t^2. Each "clump" should correspond to different numbers of hex nuts in the canisters.

Question: Do there appear to be any "missing clumps" in the v_t^2 vs. N histograms? If so, what do you suppose this means? Even if you had no apparent gaps, what would such a gap imply?

5. Using the data analysis software, create separate plots of v_t^2 vs. N for the sinkers and for the floaters. (As before, the graphs should be fairly straight lines.)
6. Using the fit routine of the analysis software, separately fit your sinker data and floater data to straight lines. From the linear fit, record the slopes of both lines.

Checkpoint: Discuss the results of your v_t^2 vs. N graphs with your in instructor before proceeding.

Instructor's OK: _________

7. Upon instruction approval, print the graphs with the slope values and include them with your lab packet. Be sure to fully annotate each graph.
8. Separately calculate the hex nut mass for the sinkers and for the floaters based on the slopes of the v_t^2 vs. N lines that you fitted with the analysis software. Don't forget the units!

   Sinker: m_{nut} = _____________ Floater: m_{float} = ____________
   ____

In the homework, you will assess how well you did. First you will measure the actual mass of the hex nuts and volume of a canister. Measure total mass of ten hex nuts, and determine the average mass of a single hex nut.

Actual mass of hex nut = __________

Measure the volume of a canister by measuring the reading of a graduated cylinder before and after a sinker is submerged in a graduated cylinder. (Be sure that the canister is fully submerged after placed in the cylinder.)

Actual volume of canister = ________

Checkout: Consult with your instructor before exiting the lab. **Instructor's OK:** ______

A MECHANICAL ANALOGY TO
THE MILLIKAN OIL DROP EXPERIMENT

Homework

Be sure to show all calculation and/or reasoning! Credit will not be given for answers that are not supported with valid explanation and/or work provided.

1. The electric charge on a particular object is found to be +1.00 nC. Have electrons been added or have electron been taken away from the object? How do you know?

2. Based on the accepted value for the magnitude of elementary charge for an electron, how many electrons have been added or taken away from the object described in the previous question.

3. Using your graphs from **Part I**, experimentally determine the point at which the sinker line and floater line intersect. *Theoretically*, what should the value of terminal speed be at this point? Conceptually, what is the significance of this point?

4. From the true mass a mass of a single hex nut, calculate a percent error separately for your sinkers and for your floaters. How well would you say your group did in measurement the "elementary mass" of the hex nut for each group?

5. Consider (separately) the slopes of your graphs for your sinkers and floaters in **Part II**. Based on the actual mass of the hex nut, were your slopes too high or too low? How do you know? What might have been the cause(s) of this?